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A COLLECTION

OF

CAMBRIDGE MATHEMATICAL EXAMINATION PAPERS;

AS GIVEN AT THE SEVERAL COLLEGES.

BY J. M. F. WRIGHT, B. A.

AUTHOR OF SELF EXAMINATIONS IN ALGEBRA,

&c. &c. &c.

PART II.

CONTAINING PAPERS IN THE BRANCHES OF THE MIXED MATHEMATICS.

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MECHANICS.

TRINITY COLLEGE, MAY 1822.

1. IF three forces acting together on a particle of matter produce no motion in it, and three straight lines be drawn making, with the directions of those forces, the same angle respectively, and each meeting the other two: then shall the sides of the triangle thus formed be proportional to the quantities of the respective forces.

2. Find generally the conditions of equilibrium on the inclined plane.

3. What force must be exerted to sustain a ton weight on a screw, the thread of which makes 150 turns in the course of 12 inches, and which is acted on by an arm 6 feet long?

4. If P sustains W on an isosceles wedge, and the wedge be put slightly into motion; prove that

$$\text{the velocity of } P : \text{that of } W :: W : P.$$

5. Explain the construction of the steel-yard.

6. In the system of pullies in which every string is attached to the weight, find the relation between the power and the weight.

7. Find the centre of gravity of any number of heavy particles placed in the same straight line.

8. If a body be suspended by any point it will not remain at rest unless its centre of gravity is in a vertical straight line drawn through that point.

9. The centres of two balls A and B move along the same straight line with the velocities a and b. Find the velocity of each after impact, when $6A = 5B$, $a = 7$ feet in a second, $4a + 5b = 0$, and *force of elasticity* : *force of compression* :: 2 : 3.

10. In uniformly accelerated motion, the whole space from rest is proportional to the square of the whole time of motion.

11. The time of falling down an inclined plane of given height is proportional to its length.

12. Twelve pounds' weight is so distributed at the extremities of a cord passing over a pulley, that the more loaded end descends through seven feet in as many hours. What weight is at each end of the cord?

13. Given the direction, and velocity of projection; find the range on the horizontal plane, the time of flight, and the greatest height attained by the projectile.

14. Find the length of a pendulum that will oscillate seconds; and of one that will oscillate four times in a second.

TRINITY COLLEGE, 1824.

1. EXPLAIN what is meant by the *moment* of a force, and shew that the moment of the resultant of two forces is equal to the sum of the moments of the components, taken with their proper signs.

2. If three forces, represented in quantity and direction by the three sides of a triangle taken in order, act upon a point, they will be in equilibrium; prove this, by resolving the forces in the direction of rectangular co-ordinates.

3. If α and α', β and β', γ and γ', be the angles which two forces make with three rectangular axes respectively, and θ be the angle which they make with each other, then

$$\cos.\theta = \cos.\alpha \cos.\alpha' + \cos.\beta \cos.\beta' + \cos.\gamma \cos.\gamma'.$$

4. A rod of given weight and length, has a moveable weight attached to it, and is placed with one end against a vertical wall, and the other upon a horizontal plane; find the position of the moveable weight, when a given sustaining force is just sufficient to prevent the rod from sliding when in a given position.

5. If l be the length of the arm of a balance, d the distance of the centres of suspension and gravity, P the load in each scale, and W the weight of the beam, then the *sensibility* of the balance is as $\frac{l}{d(2P+W)}$, and its *stability* as $d(2P+W)$.

6. When wheels act by teeth working in one another, the force of one upon the other will remain constant, if the line which is drawn perpendicular to the surfaces of both the teeth at the point of contact, pass continually through the same point of the line which joins the centres of the wheels.

7. Investigate the conditions of equilibrium upon the screw.

8. A uniform beam AB of given weight, is moveable round a hinge at A, and is kept in a given position by a weight P acting by means of a string passing over a pulley at C, and attached to the beam at B; find the weight P, when the points A and C are in the same vertical line.

9. Investigate the differential expressions for the co-ordinates of the centre of gravity of a plane curvilinear surface.

10. When a body, or system of bodies is in equilibrium, its centre of gravity is at the highest or lowest points; prove this principle, and apply it to determine the conditions of equilibrium of two bodies connected by a string, and supported upon a double inclined plane.

11. A rod of given length rests with one end upon the concave surface of an inverted paraboloid, and passes over a point which is in its focus: find the positions in which it rests.

12. A rectangular plane surface $ABCD$, is moveable round a point at A, and rests against a point at B; from the requisite data, to find the pressure on B.

13. Find the centre of gravity of a frustum of a cone.

14. If A communicate velocity to B, through a number of other bodies which are geometric means between A and B, find the limit to which the velocity of B will continually approach when the number of means is continually increased, the bodies being perfectly elastic.

15. Investigate the following equations, where v is the velocity, t the time, s the space, and f the accelerating force:

$$(1).\quad v = \frac{ds}{dt}. \qquad\qquad (2).\quad f = \frac{dv}{dt}.$$

16. Two equilateral triangles are placed with their bases at a given distance from each other upon the same horizontal line, and a

non-elastic body falls down the side of the first, along the space between the bases and up the side of the second triangle, the vertex of which it just reaches; given the side of the first triangle, to find that of the second, and likewise the whole time of the motion.

17. P draws up Q by means of a single moveable pulley, when the strings are parallel: find the force accelerating P's descent.

18. The time down a small arc of a circle to the extremity of a vertical diameter is less than the time down the chord.

19. To find the line of quickest descent from a given point to a given inclined plane, and the time of describing it.

20. A body rolls along the curve of an inverted cycloid, descending from the highest point; find the pressure upon the curve at the lowest point.

21. Two bodies, A and B, are projected at the same time from the same point with velocities a and b, one vertically, and the other at an angle of 30°; find the path described by their centre of gravity.

22. If perfectly elastic balls be let fall from the directrix of a parabola whose axis is vertical, and be reflected from the curve, it is required to find,

(1). The locus of the vertices of the parabolæ described.

(2). The locus of the extreme ranges upon the tangents at the points of incidence of the reflecting curve.

TRINITY COLLEGE, 1825.

1. If the sides $a, b, c \cdots n$ of a polygon represent the magnitudes and directions of (n) forces, which keep a point at rest, prove that $n^2 = a^2 + b^2 + c^2 + \cdots - 2(ab \cos. a, b + ac \cos. a, c + bc \cos. b, c + \cdots)$.

2. (1). Two forces which act in different planes cannot have a resultant.

(2). Given the magnitudes and directions of three forces acting in different planes upon a point; find the magnitude and direction of the resultant.

3. A given weight is supported on two given props which stand at a known distance from each other on a horizontal plane, and which are prevented from sliding, by means of a cord connecting their lower extremities; required the tension of the cord.

4. Any number of parallel forces, and the co-ordinates of their points of application being given, to determine the co-ordinates of the centre of the forces.

5. A given force being applied at a point P within a tetrahedron, it is required to decompose it into four others applied at the angular points A, B, C, D; and thence to prove, that if the lines drawn from these points through P meet the opposite faces in a, b, c, d respectively

$$\frac{Pa}{Aa}+\frac{Pb}{Bb}+\frac{Pc}{Cc}+\frac{Pd}{Dd}=1.$$

6. Shew that in any polygon, the sum of the squares of the distances of the centre of gravity from the angular points is the least possible.

7. Find the centre of gravity of a given frustum of a paraboloid.

8. Required the solidity of the greatest cone of given diameter which can be supported on a plane whose inclination is 30°; and find the highest point in the slant side where a weight may be placed without overturning it.

9. A given beam of uniform thickness and density is supported on two planes of known inclination, to find the position in which it will rest.

10. (1). When a system is in equilibrium, the centre of gravity is the highest or the lowest possible: hence,

(2). Determine the position in which two given beams of uniform density will rest, when they are placed on a horizontal plane with their lower extremities opposed to each other, and their other extremities supported on two parallel vertical planes.

11. In toothed wheels, the moment of P about the centre of the first wheel is to the moment of W about the centre of the second wheel, as the perpendiculars from the centres of the wheels upon the line of direction of their mutual action.

12. A homogeneous elliptical spheroid rests on its smaller end,

in a concave hemisphere; to find what the radius of the hemisphere must be that the equilibrium may be stable.

13. (1). Does the force of friction observe any known law? Is the quantity of friction affected by an increase of velocity, or by the extent of the surfaces that are in contact?

Explain how friction may be measured, and mention the chief contrivances that are employed to diminish it.

(2). Prove that the friction of a rope, which is wound round a cylinder, increases in geometrical progression, while the number of turns increases in arithmetical progression.

14. Two bodies P and Q are connected by a string passing over a fixed pulley; P descends vertically, and draws Q along the horizontal plane; find the space described and the velocity acquired by P in t''.

15. If a body be projected vertically upwards or downwards with a velocity (u), and be acted upon by a constant force (f), prove that

$$s = tu \mp \tfrac{1}{2}ft^2.$$

16. The axis of an equilateral hyperbola being vertical, prove that the times of descent from the extremities of the axis to any point in the curve are equal.

17. (1). The position of a ball on a triangular billiard table being given; it is required to shew that there are three directions, in any one of which, if the ball be struck, it will pursue continually the same path after being twice reflected from each of the sides.

(2). Find the position of the ball, and the direction in which it must be struck so that it may pursue continually the same path after being *once* reflected from each side.

18. A shell being discharged at a given angle, the sound of its explosion was heard at the mortar, t'' after the discharge; required the horizontal range.

19. Let A, P be two points in the same vertical plane, join A, P meeting in Q an inverted cycloid AQB, whose base AB is part of the horizontal line AX; join QB, parallel to which draw the line PC meeting AX in C: then AC is the base of an inverted

cycloid, through which a body will pass from A to P in the shortest time possible.

20. PQ is a vertical line terminating in a hard horizontal plane at Q; a perfectly elastic ball being dropped from P meets another perfectly elastic ball rebounding with a known velocity from Q, and both are reflected back; to find where they must meet, in order that they may thus rebound from one another continually.

21. A body is projected from a given point with a given velocity; to find the direction that it may just touch a given plane.

22. From what height must a perfectly elastic ball be dropped on the convex surface of a given hemisphere, so that after reflection it may describe the greatest possible horizontal range?

23. Prove that the oscillations caused by the addition of a small weight to either scale of a balance are isochronous; and show that the isochronous pendulum is equal to the diameter of the circle which passes through the fulcrum, and the points of suspension.

TRINITY COLLEGE, 1826.

1. WHAT are the objects of the science of Mechanics?

2. What are the properties attributed to matter in this science? Show the fallacy of the reasoning by which it has been attempted to prove that infinite divisibility is essential to matter.

3. What is weight? Under what limitation may it be taken as a measure of mass?

4. What kind of machine is an oar?

5. How is a wedge shown to be a species of inclined plane?

6. What is the relation of the power and weight in the second system of pullies, where the same string passes round them all?

7. What is the most effective direction for the power on an inclined plane?

8. In what case will a body of a given form be supported, or fall, on a horizontal surface?

9. Find the centre of gravity of a plane triangle.

10. Show that the times of falling down all chords of a circle to the lowest point are the same.

11. Investigate the curve described by a projectile in vacuo.

12. A hollow parabolic conoid being placed with its vertex downwards, find a point at a given height from whence a perfectly elastic ball being let fall, shall after one rebound hit the vertex.

TRINITY COLLEGE, 1827.

1. If three forces acting on a point keep one another at rest, any two are to one another inversely as the sines of the angles which their directions make with that of the third force.

2. Find the proportion between the power and the weight in the screw.

3. Prove the principle of virtual velocities in the case of the lever.

4. A body rests with its base on a horizontal plane; to find when it will be supported.

5. To find the centre of gravity of any curvilineal area.

6. If a heavy body be suspended from any point, it can only be at rest when its centre of gravity is the highest or lowest possible. Point out the different nature of the equilibrium in these two cases.

7. State the three laws of motion and the evidence on which they rest.

8. Prove that $s = \frac{ft^2}{2}$, and $v^2 = 2fs$.

9. Determine the velocities after impact of two elastic bodies impinging directly on each other with given velocities.

10. Prove that the curve described by a projectile is a parabola, and that the velocity at any point is that acquired by falling from the directrix.

11. Find the direction in which a body must be projected from a given point with a given velocity, so as to hit a given mark: and shew that there are two different directions which will solve the problem. Also find the time of flight, and the greatest altitude of the projectile above the horizontal plane.

12. The velocity acquired in falling down a system of inclined planes on the supposition that no velocity is lost in passing from one to another is equal to that acquired in falling down the perpendicular height of the system. How is this proposition to be extended to the case of a curve surface?

TRINITY COLLEGE, 1827.

1. If the resultant both of the forces p and m, and also of p and n be in the direction of the diagonals of the parallelograms, the sides of which are proportional to these forces; then will this also be true for the resultant of the forces p and $m+n$.

2. A point in the vertex of a right-angled triangle is solicited by a number of forces represented in magnitude and direction by lines drawn to equidistant points in the base; required the magnitude and direction of the resultant.

3. Given the magnitudes of three forces acting on a point, and not in the same plane, prove that the resultant is the diagonal of a parallelopiped of which these are the edges; also find its magnitude and the angles which it makes with the components.

4. If two parallel forces act in opposite directions, determine the magnitude and point of application of the resultant. What will be the result in the case when the two forces are equal?

5. A beam of wood of given weight rests with one end on the ground, and with the other on an inclined plane; what is the force necessary to prevent the plane from moving?

6. AC, CB are the equal arms of a straight lever whose fulcrum is C: to C a heavy arm CD is fixed perpendicular to AB. Prove that when different weights are suspended from the extremity A, the tangents of the inclinations of CD to the vertical will be proportional to the weights.

7. Find the centres of gravity

(1). Of a triangle, the density of which in every part is as its distance from the base.

(2). Of the area of the logarithmic curve intercepted between two ordinates.

8. Two heavy spheres, of given dimensions, are placed in a hollow hemispherical basin. Determine their position of equilibrium.

9. If two parallel forces act in the same direction on the opposite angles A and C of the parallelogram $ABCD$, and a third force act on the point B in the direction of the diagonal BD; find the magnitude and point of application of a fourth force which will keep the parallelogram at rest.

10. In what direction must a ball be projected along the interior of a hollow spherical superficies, so that it may pass through a given point? the ball being supposed to be without weight.

11. A number of balls of given weight are projected at the same instant in given directions with given velocities; find the height of their common centre of gravity after a given time, and the highest point to which it will rise.

12. Water is to be raised in a bucket from a well of given depth by means of a given weight hanging over a fixed pulley. What weight of water must be raised each time, so that the greatest possible quantity may be raised in twelve hours?

13. P draws Q up a groove cut out in an inclined plane; find the velocity of Q at any point, the angle which the string makes with the plane varying at every point.

14. OA and OB are the vertical and horizontal radii of a circle; it is required to find a point C in the quadrant AB to which if a tangent be drawn meeting the radius OB produced in D, and a line touching the circle at A in the point E, the time down DE + time of moving along EA with the acquired velocity may be a minimum.

15. From the highest point A of the vertical diameter AB of a circle, draw the line of quickest descent to the cissoid of which B is the origin and AB the axis.

16. A and B are two balls of given elasticity; what must be the magnitude of a third ball, that the velocity communicated from A to B by the intervention of this ball, may equal that communicated immediately from A to B? Determine also the limits within which the problem is possible.

17. Two balls are projected at the same instant from two given points in a horizontal plane, and in opposite directions, so as to de-

scribe the same parabola. What must be their relative magnitude, and their elasticity, so that after impact one of them may return through the same path as before, and the other descend in a right line?

18. A body moves in a cycloid the plane of which is inclined to the horizon at a given angle; find the time of an oscillation, and the point where the velocity perpendicular to the horizon is a maximum.

TRINITY COLLEGE, MAY 1828.

1. IF two weights acting perpendicularly upon a lever on opposite sides of the fulcrum, have their distances from the centre inversely as their weights, they will balance each other.

2. Enumerate the definitions and axioms which are requisite in the proof of the preceding proposition.

3. Four weights, 1, 3, 7, 5, are at equal distances on a straight lever. How far from each is the fulcrum on which they will balance?

4. In the system of pullies in which the same string passes round all the wheels, what is the proportion of the power and weight?

5. Explain its construction that all the wheels may revolve in the same time. [White's Pully.]

6. Explain any contrivance by which a reciprocating motion upwards and downwards may be converted into a continued circular motion.

7. What is the use of a *fly-wheel*? How would the defect be perceived of its being too heavy or too light?

8. Explain and prove the second law of motion. Show from it that a projectile will describe a parabola.

9. On what assumptions can the third law of motion be deduced from the second? Are these assumptions allowable?

10. Explain the principal peculiarities in the different mechanical agents or first movers which we can employ: *gravity, water, air, heat, animal strength.*

TRINITY COLLEGE, MAY 1828.

1. ASSUMING that the resultant of two forces is *in the direction* of the diagonal of the parallelogram, show that its *magnitude* is proportional to the diagonal.

2. If a piece of wood be dropped into a running stream, show that it can never accurately acquire the velocity of the stream.

3. If three forces keep a point at rest, they are each inversely as the sine of the angle contained by the other two.

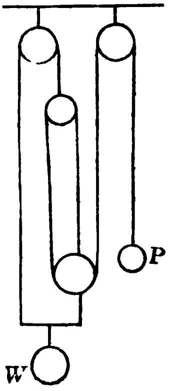
In the last question let the angles be 135°, 120°, 105°: what is the proportion of the forces?

4. In rowing, suppose the oar to be 12 feet long: the handle moves *forward* with the velocity of 11 feet, the extremity of the blade moves *backward* with a velocity of 1 foot; the row-lock is $2\frac{1}{2}$ feet from the handle. Compare the *pull* of the rowers with the force exerted against the water by the boat.

5. In the annexed system of pullies, compare P and W.

In this system of pullies, show that

$P : W :: W$'s velocity : P's velocity.



6. Out of a square, a triangle is to be cut, having one side for its base, so that the centre of gravity of the remaining portion may be at its vertex.

7. The sum of the products of each particle into the square of its distance from the centre of gravity, is less than the same sum for any other point.

8. If in a two-wheeled carriage the centre of gravity be *before* and *below* the axle-tree, compare the pressure on the horse on a horizontal, ascending and descending road.

9. On the inclined plane what is the relation between the force and weight in the case of equilibrium?

In this case if friction be $\frac{1}{3}$ of the pressure, what are the limits within which the power may vary without having the equilibrium destroyed?

10. Two unequal spheres of the same material are placed together in a hemispherical bowl; find the position of their points of contact with the bowl.

11. Any number of forces being given, to reduce them to two sets of forces, one set being in a given plane, and the other perpendicular to it.

12. A regular tetrahedron $ABCD$ is acted on by forces in the directions of its edges, viz. AB, AC, AD, BC, CD, DB: reduce these forces to two, and show that the reduction to one is impossible.

13. Find the centre of gravity of a figure bounded by the arc of a parabola, its directrix, and two lines parallel to the axis.

14. Four beams of equal length and weight are placed so as to form a regular roof: having given their position, find the pressure in the direction of each, and the weight which will be supported at each angle.

15. An umbrella has eight straight ribs: supposing the ends of these to be connected by strings, compare the force which pushes the slider along the stick with the force which stretches these strings.

16. A picture suspended by means of a string fastened to two points in its upper edge and passing over a nail, hangs *askew:* has it a tendency to slip to or from the horizontal position, and what friction on the nail will resist this tendency?

17. Prove the expressions $s = \frac{1}{2}ft^2$, $v^2 = 2fs$.

18. If a body fall through a distance a at two different places, and if the time of falling at one of them be t'' less, and the velocity acquired m feet greater than at the other, compare the force of gravity at the two places.

19. In an inverted parabola the time of descending down any chord from a point P to the lowest point is equal to the time of descending vertically to a horizontal line which is at a distance below the vertex equal to the latus rectum.

20. A perfectly elastic body is projected from a point in an inclined plane: α being the angle at which it must be projected so that after striking the plane it may be reflected vertically upwards,

$$\tan.\alpha = \frac{3\tan.\iota + \cot.\iota}{2}.$$

21. A beam is 1 foot square: a string is wrapped 3 times round it, and hangs 3 feet below the beam with a weight at its end: what horizontal velocity must be given the weight that it may entirely unwrap the string?

22. The time of falling down any portion of a cycloidal arc is $\frac{\theta}{2}\sqrt{\frac{4a}{g}}$; where a is the radius of the generating circle, and θ the angle obtained by describing a circle on the original abscissa and cutting off an arc of it by a horizontal line passing through the place of the body.

23. If the ends of a bow be always at the same distance from the middle, and the string always equally stretched, show that the time of the arrow coming up to the bow is the same, however far the bow be drawn.

TRINITY COLLEGE, June 1829.

1. Enumerate the various kinds of levers.

2. Explain the constructions of the common steel-yard, and of the weighing machine used at toll-houses.

3. What contrivances are used to convey rotatory motion with a change of direction?

4. Describe the various kinds of crank used in machinery.

5. What are the circumstances under which force is required to be applied in printing? How is the purpose effected in the Stanhope Press?

6. Describe the contrivances used to produce *parallel motion* in machines, and assign the proper relation between the parts in connexion.

7. Describe the construction of a roof with a tye-beam, king-post, and queen-post. What are the uses of these?

8. What is the supposition made in the common theory of the arch? How does it appear to be insufficient?

9. Explain the method of finding the position of the centre of gravity of any number of given connected material points. Apply it to find that of a triangular lamina of uniform thickness.

10. Explain the method of indivisibles, and show that, when incautiously used, it may lead to erroneous results in questions like the last.

11. In the direct impact of two perfectly inelastic bodies A and B, estimating the effects in the direction of A's motion,

$A + B : A$ :: relative velocity : velocity gained by B.

12. If chords be drawn in a circle from the extremity of that diameter which is vertical, the velocities which bodies acquire by falling down them are proportional to their lengths, and the times of descent are equal.

TRINITY COLLEGE, June 1829.

1. Assuming that when two forces, acting simultaneously on a point, are equal, the direction of the third force which balances them bisects the angle between the directions of those two forces, prove in general that the force which balances two other forces is represented, in direction and in magnitude, by the diagonal of the parallelogram, the two adjacent sides of which represents the individual forces.

2. Define the term "moment of a force:" and prove that the sum of the moments of any number of forces, acting on a point, is equal to the moment of their resultant.

3. How does the force requisite to draw a carriage wheel over a small obstacle vary, supposing the weight of the carriage supported on the axis of the wheel?

4. Explain the principle of the graduation in the common and in the Danish Steelyard.

5. Determine the ratio of the power to the weight in the first system of pullies, including the weights of the blocks.

6. A mass of a given weight being placed on an inclined plane determine from the requisite data,

(1). the force which will just prevent it from sliding downwards,

(2). that which just set it in motion upwards.

7. Investigate the conditions of equilibrium in the wedge, and show what mechanical advantage is gained by using that instrument as it is commonly employed.

8. Prove that the altitude of the centre of gravity of any pyramid is one fourth of that of the pyramid itself, the base being supposed horizontal.

9. A given weight Q hanging over a pulley balances another weight P resting at any point of a curved surface; required the form of the surface.

10. Find the equation to the catenary between x and s, beginning at the lowest point.

11. Describe briefly the experiments made by Attwood, with the machine invented by him, to verify the laws of motion.

12. Distinguish between accelerating force and moving force.

13. Explain the proposition or axiom: Reaction is equal and opposite to action.

14. In the collision of two masses A and B, having an elasticity which is to perfect elasticity as $1 - \epsilon : 1$, determine the velocities after impact, and the *vis viva* lost.

15. Investigate the equations of motion,

$$v = \frac{ds}{dt}, \ f = \frac{dv}{dt}.$$

16. A mass P hanging over a pulley draws another Q horizontally along a plane surface. The friction of Q being $\frac{1}{n}$th of the pressure, determine the accelerating force on P.

17. A material point being projected in vacuo with a given velocity in a given direction, determine,

(1). Its place at the expiration of any given time from the beginning of its motion;

(2). The nature of the path which it describes.

18. When a mass is projected directly upwards or downwards from a given point with a given velocity, the equation from which the time of its reaching any given height is to be determined, being

a quadratic equation, gives two values for the time. Explain these in all the different cases.

19. Determine the time of oscillation in a cycloidal arc.

20. Explain the contrivance used by clock makers to assimilate the motion of a common pendulum to that of a cycloidal one.

21. A pendulum, which should beat seconds, is found to lose 10″ a day. Determine the quantity by which it should be lengthened or shortened.

22. Explain the principle of the mercurial compensation pendulum.

TRINITY COLLEGE, MAY 1830.

1. If two forces acting at any angles on the arms of any lever are inversely as the perpendiculars from the fulcrum upon their directions, they will balance each other.

2. Show how to graduate the common steel-yard.

3. Describe Smeaton's system of pullies, and mention its peculiar advantages.

4. Required the ratio of the moment of P to the moment of W in toothed-wheels.

5. If the equilibrium in the single moveable pully with strings not parallel be disturbed in a small degree, P's velocity will be to W's as W to P.

6. Find the centre of gravity of any number of bodies P, P', P'', &c. considered as points in the same straight line.

7. From the first and second laws of motion, infer what the effect will be of communicating to any mass given velocities in two different directions at the same instant.

8. If impact take place between two imperfectly elastic bodies moving in the same straight line, the momentum gained by one, will be equal to that lost by the other.

9. In what manner are velocity and momentum, accelerative force and moving force, numerically estimated?

10. Prove that if the accelerative force be constant.

$$v = ft, \text{ and } s = \frac{ft^2}{2}.$$

11. The time of a small oscillation in a circular arc is very nearly independent of the extent of the oscillation.

12. What is the principle of the *dead beat* scapement, by which the oscillations of a clock pendulum are maintained, without destroying the isochronism?

TRINITY COLLEGE, MAY 1830.

1. If three forces keep a point at rest, they are each directly as the sine of the angle contained by the other two.

2. Without assuming the properties of the lever, determine the quantity and direction of a single force, equivalent to two parallel forces; (1) when they act in the same direction; (2) when they act in opposite directions.

3. When four parallel forces, consisting of two *couples* of equal and opposite forces, act in the same plane on any system, and keep it at rest, the couples tend to *twist* in opposite ways, and the distance between the lines in which the forces of each couple act, is inversely proportional to the magnitude of the forces.

4. Required the proportion of the power to the resistance in the wedge.

5. Having given the dimensions of a cone placed with its base resting on an inclined plane, and the ratio of friction to the pressure, determine whether, upon gradually increasing the inclination of the plane, the cone will tumble or slide.

6. When a weight is supported on an inclined plane, by means of another with which it is connected by a string passing over a fixed pully, if a small motion be given to the system, the centre of gravity of the weights will neither ascend nor descend.

7. Given the magnitudes and directions of two forces, P, Q, acting at the extremities of a rigid rod AB; required the magnitude, direction, and point of application of a single force R, which will just counteract the other two.

8. If any number of forces P, P', P'', &c. acting on any rigid body in the same plane, keep it at rest, and from any point in that plane perpendiculars p, p', p'', &c. be let fall on their directions, then will

$$Pp + P'p' + P''p'' + \&c. = 0.$$

9. Prove the principle of virtual velocities by means of Lagrange's *principle of pullies.*

10. Apply the principle of virtual velocities to find the position of rest of a uniform beam of given length, pressing with its extremity against a vertical wall, and leaning on a prop situated at a given distance from the wall.

11. Find the centre of gravity of any curvilinear body.

12. Enunciate Guldinus's Properties, and by means of them find the centres of gravity of a semi-circular area, and a semi-circular arc.

13. If any area revolve round an axis, lying in its plane, and dividing it into any two parts, the *difference* between the solids generated by the parts, will be equal to the whole area multiplied by the path of its centre of gravity.

14. A chain of given weight w and given length l, is suspended between two fixed points in the same horizontal line, distant from each other by a; supposing $l - a$ to be very small compared with l, and the form of the chain to be nearly a circular arc, an approximate value of the tension at the lowest point is

$$\frac{wa}{2l}\sqrt{\frac{a}{6(l-a)}}.$$

15. Show from the common theory of the arch, that if the height of an arch be great compared with its span, it requires a great weight at its crown, to maintain the equilibrium. What is the reason that this result of the common theory, which neglects friction, accords with what is the case in practice?

16. The direction and velocity of the motion of the centre of gravity of two bodies, is not altered by their impact.

17. Required the range and time of flight of a body projected with a given velocity in a given direction, at the foot of a plane inclined at a given angle to the horizon.

18. Prove that when a body oscillating in a cycloid, receives a sudden accession of velocity at a certain point, before arriving at the lowest point, the time of its oscillation is as much diminished, as it would be increased, if the body received an equal accession of velocity at the same distance from the lowest point, after leaving it.

19. Obtain a series for the time of oscillation in a circle.

20. Define the *moment of inertia* of a revolving system, and prove the following proposition: the moment of inertia of any system, with respect to any given axis, is equal to the moment about an axis parallel to this, passing through the centre of gravity, *plus* the moment of the whole body (collected in its centre of gravity) about the given axis.

21. The centre of suspension and oscillation are reciprocal.

22. Two *equal* cylinders, connected together by a string, which winds round both and passes over a fixed pulley, are suffered to descend vertically by unrolling from the string; it is required to find the accelerative force of each, and the tension of the string.

23. A point moving in any manner in space, has its position with respect to a fixed point determined at a time t, by the co-ordinates x, y, z; and it is acted upon by the forces X, Y, Z, in the directions of x, y, z, respectively; shew that

$$X = \frac{d^2x}{dt^2}, \quad Y = \frac{d^2y}{dt^2}, \quad Z = \frac{d^2z}{dt^2}.$$

In the proof make it appear that these equations have become known in consequence of observation at the Earth's surface of the fact, that gravity communicates the same velocity in the same time in the direction in which it acts, to any body, moving in any direction whatever and with any velocity.

ST. JOHN'S COLLEGE, JUNE 1821.

1. IN a system of pulleys where each string is fixed to the weight, and the strings are parallel, shew that $W = (2^n - 1) P$. If the weight of the pulleys be taken into the account, is the mechanical advantage increased or diminished?

2. AC, BC are two inclined planes meeting a horizontal plane at the same point C. Having given the inclination of the planes, and the length of BC, find the point A such that a body descending down AC may just ascend to the top of the plane CB.

3. A person ascending in a balloon, lets fall a stone when at a given height. Find the time (t) of the stone's reaching the ground, supposing the velocity of the balloon at the given altitude known; and explain the meaning of the negative value of (t).

4. The compressing force is to the force of elasticity :: 3 : 1. At what angle must such a body be incident of a hard plane, that the angle between the directions before and after impact may be a right angle.

5. *APC*, *AWB* are two equal quadrants connected at *A* and having their planes vertical, *P* and *W* are two weights connected by a string passing over a pulley at *A*, the string *AP* passes along the curve through a groove while *W* hangs freely. Having given the position of *P*, find the position of *W* when there is an equilibrium.

6. A circle has chords drawn from the extremity of its diameter which is at right angles to a horizontal plane. Find that chord down which an imperfectly elastic body descending and reflected by the horizontal plane may describe the greatest horizontal range, and knowing the degree of elasticity, find the range.

7. A string, having its extremities fixed to the end of a cylindrical bar of uniform density whose weight is known, passes over four tacks so as to form with the bar a regular hexagon, the bar being horizontal. Find the tension of the string, and the vertical pressure on each tack.

8. A beam *AB* of given length and weight, rests with one end on a given inclined plane, and the other attached to a string *AFP* passing over a pulley at *F* given in position. Knowing the weight *P* fixed to the other end of the string, find the position in which the beam rests.

9. *AVB* is a cycloid with its axis *DV* vertical, and vertex *V* downwards, *EOF* parallel to the base, and bisecting the axes in *O*. At *E* and *F* are two vertical hard planes. Show that a body descending from *E* with the velocity acquired in falling down a vertical space equal to *EO*, will by being reflected at *F* and *E* continually describe the same path; and find the interval between its leaving *E* and returning to it.

10. A body suspended by a string oscillates on an inclined plane through a semicircle whose diameter is horizontal; find the tension of the string at any point, and the law of its variation at any point for different inclinations of the plane.

11. Shew how the following equations for determining the

motion of a body acted on by a constant force are derived

$$2S = FT^2 = 2FS, \quad V = FT,$$

and explain what each of the letters S, T, V, F, represent.

12. If the interior of two circles which touch internally be taken away, and the remainder vibrate edgeways round an axis passing through the point of contact, find the centre of oscillation.

13. A body descends down the arc of a parabola, whose plane is vertical and axes horizontal, and is acted upon by gravity and a repulsive force tending from the focus which varies as $\frac{1}{(\text{dist.})^2}$. Find the velocity at any point.

14. If a body describe from rest any part of a curve AB on an inclined plane, the velocity at B will equal the velocity acquired in falling through a vertical space equal to the perpendicular depth of B below A, and the time of describing AB on the plane : time of describing AB (were the plane vertical) :: $\sqrt{}$(sine inclination) : radius.

15. If any weights A, B, C, act upon a machine, and put it in motion, and x, y, z be the spaces described in the direction of gravity, a, b, c the actual velocities of the weights, $m = 16\frac{1}{12}$ feet, then

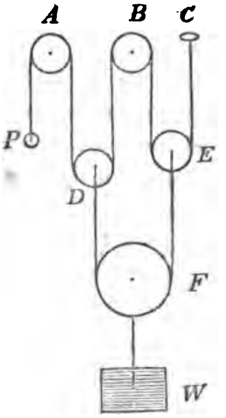
$$4m(Ax + By + Cz) = Aa^2 + Bb^2 + Cc^2.$$

16. Apply the above proposition to find the greatest angular velocity that can be acquired by the lever ACB whose arms CA, CB are equal and at right angles to each other, CA being at first horizontal, and the density of the arms varying as the distance from C.

ST. JOHN'S COLLEGE, MAY 30, 1822.

1. IF two weights balance each other on a straight lever in one position of the lever, they will balance in all positions. How far is this true of a bent lever?

2. There are n weights a, b, c, &c. in geometric progression, and (a) placed at A one extremity of a lever balances (b) placed at B, the other extremity. Prove that a weight = the $n - 1$ first weights if placed at A will balance a weight = the $n - 1$ last weights at B.



3. In the annexed system the strings are parallel, the pulleys *A* and *B* are fixed, and *D, E, F* are moveable. Find the ratio of *P* : *W*, and the pressures at *A, B, C,* when there is an equilibrium.

4. Given that the distance of the centre of gravity of a cone from the vertex is $\frac{3}{4}$ths of the axis, find the centre of gravity of a conic frustum, the radii of whose bases are *R* and *r*, and altitude *h*.

5. *P* and *W* are weights fixed to the ends of a given circular arc, which is placed with its plane vertical on a plane inclined to the horizon at an angle (θ). Shew that it will rest with the chord parallel to the plane, when

$$P : W :: a - b \tan.\theta : a + b \tan.\theta ;$$

(a) being the sine and (b) the versed sine of half the arc.

6. *ADB* is a semicircle, *AB* the diameter, *DC* the sine of *BD*. An inelastic body moving uniformly along *AD*, impinges on the plane *CD*; prove that it will move uniformly along *DC*, and that the time along *DC* will vary as tan.*BD*.

7. Define uniform force, and prove that the space described from rest by a body acted on by an uniform force varies as the square of the time; and point out in the proof where it is assumed that the force is uniform, and that the space is reckoned from the beginning of the motion.

8. The sides *AB, CD* of a billiard table are parallel, and an imperfectly elastic ball struck from a point *C* in one side, impinges at *E* in the other, and is reflected to *D* in the first side. Shew that time along *CE* : time along *ED* :: force of elasticity : force of compression.

9. With what weight must a given weight *P* be connected by a string passing over a single fixed pulley, so as to describe the same space in a given time as when it descends freely down a given inclined plane?

10. *BDC* is a semi-cycloid with the axis *AC* vertical; *FDE, CE* are tangents at *D* and *C*. Find *D*, so that the time down *FE* +

time along *EC* with the velocity acquired down *FE* may be a minimum.

11. If an inelastic body move uniformly along one side of a regular polygon, shew that it will continue to describe the other sides uniformly, but with velocities decreasing in geometric progression; and, in the case of a hexagon, shew that the time of describing the first side : time of describing the last :: 1 : 32.

12. If a body be projected up a plane *AC* inclined at 45° to the horizon with the velocity acquired in falling down a vertical space $= AC$, find the range *AD* on the horizontal plane passing through the point *A*. Determine also the time between its leaving the point of projection, and meeting the horizontal plane.

13. When *W* is raised by *P* in the wheel and axle, find the pressure on the axis, neglecting the inertia of the machine.

14. If two bodies whose weights are as 2 : 3, oscillate one in a semicircle, and the other in a cycloid, the whole tensions of the two strings at any given inclination of them to the horizon will be equal; the motion in each case beginning from the highest point.

15. The points *A*, *B*, *C*, &c. are taken equidistant from each other in the vertical line *AS*. A body half elastic, beginning its descent from *A*, impinges on a perfectly hard horizontal plane at *B*, and in its second descent against another at *C*, and so on. Find the space described between the $(n-1)$th, and nth rebounds, and the time of describing it.

16. *AB*, *CD* are two vertical planes, *BD* is horizontal. A perfectly elastic ball projected from *A* is reflected by the planes *CD* and *BD* to *A*. Find *BD*; having given *AB*, and the velocity and direction of projection.

17. Two equal bodies begin their descent at the same instant down a plane inclined at 30° to the horizontal plane, and in directions perpendicular to the common section of the two planes. One of them after descending half way is reflected by a perfectly elastic plane inclined at 45° to its course. Find the nature and dimensions of the path traced out by their common centre of gravity.

18. In a system of (n) equal wheels and axles, where *W* is raised by a power applied at the circumference of the first wheel; find the force accelerating *W*, and the tension of the string to which *W* is attached; *w* being the weight of each wheel with its axle, *d* the

distance of the centre of gyration from the axis, and $r : 1$ the ratio of the radii of each wheel and axle.

19. AB is the diameter of a circle, C the centre, CD perpendicular to the plane of the circle, which vibrates about an axis passing through D parallel to AB. Find the centre of oscillation.

20. Supposing the above circle to move parallel to itself, with its centre in the line CD, and thus generate a solid; find the locus of A and B, that every section of the solid perpendicular to CD, may oscillate in the same time as a pendulum whose length is (l), the axis of suspension remaining the same.

St. JOHN'S COLLEGE, 1824.

1. Explain how the densities and magnitudes of bodies, and the quantity of matter they contain, are numerically expressed, and from thence deduce the equation, which shows the relation subsisting between them.

2. From what experiments is it inferred, that when the same force is impressed upon different bodies, it communicates to them velocities, which are inversely proportional to their quantities of matter, and how do you reconcile this law with the fact, that the force of gravity, which is every where the same, communicates the same velocity to bodies of different magnitudes?

3. Action and re-action are equal, and in opposite directions. Explain fully what is meant by action, and what by re-action, and illustrate your explanation by stating the experiment, which in Wood's Mechanics is advanced as a proof of this law.

4. AC and BD are two beams, whose weights are given, moveable in a vertical plane about the fixed points A and B, BD rests upon AC as a prop. Find the position of equilibrium.

5. An imperfectly elastic body descending vertically from rest, meets an horizontal plane, which is moving uniformly in an opposite direction; having given the distance between the plane and the body at first, and the degree of elasticity, find what must be the velocity of the plane, so that the body may return to the point, from which it fell.

6. If perpendiculars be drawn from any number of bodies to a

given plane, the sum of the products of each body, multiplied by its perpendicular distance from the plane, is equal to the product of the sum of all the bodies multiplied by the perpendicular distance of their common centre of gravity from the plane.

7. Having given the height and the breadth of a house, find the equation, from which may be determined such an elevation of its roof, that a body, beginning to descend from its summit, may fall upon the ground at the greatest possible distance from the bottom of the house.

8. Explain what is meant by a state of stable and of unstable equilibrium, and supposing a semi-ellipsoid to rest upon its vertex, what must be the ratio between its major and minor axes, that it may just rest in a state of stable equilibrium?

9. S and S' are two centres of force varying as $\frac{1}{(\text{dist.})^2}$, and their intensities are equal. A body begins to descend from a point, which is at an equal distance from them both. Find its path and velocity at any point.

10. Suppose (as in the previous question) that the distance of the point, from which the body begins to fall, had been at an inconsiderable distance from the straight line joining S and S', find the time of the body's oscillation.

11. To find the centre of oscillation of a cube, vibrating about an axis coincident with one of its edges.

12. A solid of revolution stands with its axis vertical. An elastic string, whose weight is given, having its two ends connected together, and thus forming a kind of flexible ring, is thrown over it, and being acted upon by gravity, descends till it rests in an horizontal position. Find its position of equilibrium, and shew also what must be the nature of the curve, that it may rest at all altitudes.

ST. JOHN'S COLLEGE, MAY 1825.

1. CONSTRUCT the common balance, and shew that if (α) be the angle at which the arms are inclined, and that P and Q when in equilibrium sustain the lever at an angle θ with the horizon,

$$\tan.\theta = \frac{P-Q}{P+Q}\cdot\cot.\frac{\alpha}{2}.$$

2. An apparent pound weight is weighed at each end of a false balance; the whole when examined is found to contain $2\frac{1}{2}$ lbs. Required the ratio of the arms.

3. A given triangle without weight rests horizontally on three props placed at its angular points; find a point in it from which, if a given weight be suspended, the pressures on the three props may be in a given ratio to each other.

4. P and W are supported on a wheel and axle, P by a string passing round the wheel, W by a moveable pulley whose strings are parallel, one of which, as P descends, winds on the wheel and the other off the axle. Required the ratio of P to W in case of equilibrium, and point out the advantage of this system.

5. Find the ratio of P to W in the single moveable pulley; and if P descends, find the curve which is the locus of W. Is the above a case of stable equilibrium?

6. Investigate the conditions of equilibrium on the inclined plane without having recourse to the composition of forces.

7. Find the quantity and direction of the force exerted by a door of given weight and dimensions on each of its hinges.

8. A quadrant being placed vertically, a chain of given weight is laid on the arc and coincides with it. Required the force necessary to keep it from sliding down.

9. Required the radius of the least sphere on which a given oblate spheroid can rest in a position of permanent equilibrium.

10. State the principle of virtual velocities, and prove it in the case of the inclined plane.

11. A row of (n) elastic balls, A, B, C, &c. is placed in a right line. Required their ratio, so that the motion of A may after impact be equally divided among the bodies.

12. An elastic sphere, after impinging on a harder plane (not smooth), is observed to move off at an angle equal to the angle of incidence. Required the sphere's elasticity.

13. A weight (w) is connected by a string passing over a fixed pulley with a number of equal balls (n), placed in a vertical cylinder whose diameter equals that of the balls. They are connected to each other by elastic threads equal and given in length. Required the velocity of (w) when (n) balls are drawn up, and the number required to destroy (w)'s velocity.

14. Find the locus of all the points to which a body will descend in the same time on a straight line drawn from two given points.

15. Find the line of quickest descent from the focus to the curve of a parabola whose axis is vertical.

16. Make a body oscillate in a given cycloid, and find the actual time of oscillation.

17. If θ be the angular distance from the lowest point of a circular arc, shew that the force in the direction of the curve is to that in the direction of the chord as $2 \cos.\frac{\theta}{2} : 1$.

18. If a ball whose elasticity : perfect :: n : 1 be projected in vacuo, at an $\angle \theta$, with velocity V; prove that the sum of all the horizontal ranges $= \frac{V^2 \sin.2\theta}{g(1-n)}$, $g =$ gravity.

19. Explain the principle of the rifle barrel.

20. Given the increase of length in a rope stretched horizontally by its own weights; find its increase of length when hung vertically.

ST. JOHN'S COLLEGE, 1826.

1. If a body be kept at rest by three parallel forces, shew that they are in the same plane, and any two of them are to each other inversely as their distances from the third.

2. The force requisite to draw a wheel over a small obstacle varies nearly as the square root of the height of the obstacle. Required proof.

3. Find the conditions of equilibrium when a body moveable about a fixed axis is acted on by any number of forces.

4. W is sustained on the circumference of a vertical circle by means of a string going over a pulley at the highest point from which P is suspended. Given the ratio of P to W, find the position of equilibrium, and determine whether it is stable.

5. Prove in the case of the inclined plane that the power : weight :: weight's velocity : power's.

6. If a force act on a body in free space, shew that the motion of the centre of gravity is the same at whatever part of the body the force is applied.

7. Find the centre of gravity of a conical glass and of a hemispherical bowl.

8. A cylinder rests on a sphere with its axis vertical. Under what limits of the base and altitude will the equilibrium be permanent?

9. Two elastic balls ($A = 3B$) are placed on a horizontal table. A impinging with a given velocity on B at rest drives it perpendicularly against a hard vertical plane, and it meets A in returning at half its original distance. Required the elasticity of the balls, and time of motion.

10. A rod of given length and weight projects obliquely from a wall: its upper extremity is sustained by two strings of given length, which are fixed to two given points in the wall. Required the tension of each string.

11. Given the weight which a beam of given weight and dimensions can sustain, find the dimensions of a similar beam of the same materials which will sustain the greatest weight.

12. Define uniform force, and *thence* deduce the equations $V = FT$, and $S = \frac{1}{2}TV$. Explain the symbols, and state whether S and V begin from 0, and why.

13. State and prove distinctly the third law of motion.

14. A *hollow* cylinder of given weight is placed with its axis horizontal. A string of given length is *fixed* to, and wound round it; and to the extremity of which P being attached is suffered to descend. Find the height to which P rises on its *ascent*.

15. Given the velocity of projection, find the direction so that the projectile may pass through a given point (without the aid of the parabola).

16. Given the point of projection, find the velocity and direction, so that the projectile may pass through a given point and strike the ground in another given point.

17. A parabola being placed with its axis vertical and vertex downwards, from what point in the extreme ordinate must an elastic ball be let fall so that after impinging six times on the curve it may return to the same point?

St. JOHN'S COLLEGE, May 1828.

1. State fully the second law of motion, and mention some of the observations by which it is established.

2. A body is moving in a given direction with a given velocity; required the direction in which another given velocity less than the former must be communicated to it, so as to make the greatest alteration in the direction of its motion. And required its new velocity and direction.

3. In the direct impact of perfectly elastic bodies, the sum of the products of the bodies, and the squares of the velocities, is the same before, and after impact.

4. Two weights P and Q, balance each other upon the surface of a given sphere by a string of given length, passing over the vertex. Required the position of equilibrium.

5. Find the centre of gravity of the arc of a semi-cycloid.

6. Required to project a body, whose elasticity is to perfect elasticity $:: m : 1$, from a given point in the circumference of a circle, so that after two reflections against the curve, it may return to the same point; and to shew that the times of describing the first and last chords, are as $m : 1$.

7. A body being projected down an inclined plane with the velocity which would be acquired in falling down its perpendicular height, the time of descent is found to be equal to that of falling down the height. Required the plane's inclination.

8. A chain hangs in the form of a parabola whose axis is vertical, each particle being acted on by a force parallel to the axis and varying inversely as the distance from the directrix. Required the law of the density.

9. Required the least velocity, with which a body must be projected, to strike a given object in the horizontal plane, supposing a wall to intervene, whose distance from the object is equal to its own height.

10. Prove that if a body acted on by gravity be descending down the convex side of a plane curve, it will leave the curve, when the horizontal velocity is a maximum; and apply it to determine when

a body moving from rest from a given point down the convex side of a cycloid, axis vertical, will leave the curve.

11. In the last Problem, determine the time elapsed.

12. Two uniform beams of given weights, and lengths, have their upper extremities against two smooth parallel vertical planes; and their lower extremities, which rest on the intermediate horizontal plane, act against each other. Required what must be the distance between the planes, when the beams rest at right angles to each other.

13. State D'Alembert's principle and apply it to find the accelerating force on an uniform plane in the form of an equilateral triangle oscillating about one side. Also determine the whole pressure on the axis in any position.

14. Given W the weight of the body of a waggon, P and Q the weight of a fore and hind wheel respectively. Find the time in which it will run down a given inclined plane, the wheels being supposed uniform solid cylinders; and shew that it will descend quicker when laden.

ST. JOHN'S COLLEGE, JUNE 1829.

1. THE force which accelerates or retards a body's motion in a cycloid varies as the arc intercepted between the body and the lowest point.

2. A sphere is sustained upon an inclined plane, by the pressure of a rod moveable about the lowest point of the inclined plane; given the position of the rod, required that of the plane.

3. Given the distances of the centres of gravity of any number of bodies from a plane, find the distance of their common centre of gravity from the same plane; and shew that its motion is uniform and rectilinear, provided the motions of the centres of gravity of the several bodies be so.

4. A lever touches with its arms, which are at right angles, the circumference of a circle, whose plane is vertical, and radius less than either of the arms; determine the position in which it will rest.

5. A body descends down an inclined plane of given height and

length, and rebounds from the horizontal plane; given the greatest elevation which it afterwards attains, determine its elasticity.

6. Three equal spheres are placed in contact upon a horizontal plane; if another equal sphere placed upon them just causes them to separate, what is the proportion of friction to pressure?

7. If two tangents to a parabola are at right angles to one another, and their point of intersection be in the axis, find the centre of gravity of the area contained between them and the arc; and determine the curve described by the centre of gravity, when the tangents sume all possible positions.

8. Find the equation to the path of a projectile; and shew that its length, between the body leaving the horizontal plane and returning to it, will be the greatest possible for a given velocity, when the angle of elevation (θ) satisfies the equation

$$2\sec.\theta = e^{\text{cosec}.\theta} + e^{-\text{cosec}.\theta}.$$

9. What segments of a sphere will oscillate about a tangent at the vertex in the shortest, and longest times possible? find the length of the isochronous simple pendulum in each case.

10. Determine the position of a weight supported on a smooth cycloidal lamina whose axis is vertical, by an elastic string which lies along the arc and is fastened at the vertex; supposing the length of the string when unstretched given, and its weight inconsiderable.

11. Two equal balls, resting on a smooth horizontal table, are fastened by two equal strings to the end of another string, which, passing over the edge of the table, sustains a given weight; determine the motion, supposing the initial direction of the latter string to bisect the angle between the two former.

12. An uniform beam rests in a vertical plane with its ends upon two curves; shew that if the angles of inclination to the horizon of the normals, at the points in contact with the beam, be respectively α and β, and θ be the inclination of the beam itself to the horizon, then $2\tan.\theta = \tan.\alpha \sim \tan.\beta$.

13. A lamina of matter in the form of an ellipse slides by gravity with its plane vertical, between two straight lines inclined at angles α, and $90 - \alpha$ to the horizon, determine the motion.

14. An uniform and flexible string is suspended from two points

in the surface of a vertical cylinder, determine its equations; taking the origin where the axis of the cylinder meets a horizontal plane, whose distance from the lowest point of the string = length of a portion equivalent in weight to the tension at that point; and shew that the perpendicular upon the tangent from the origin is of a constant length.

ST. JOHN'S COLLEGE, MAY 1830.

1. IF three forces keep a point at rest, any one of them is proportional to the sine of the angle between the directions of the two others.

2. A stone dropped from a bridge, strikes the water in $2\frac{1}{2}''$, what is the height of the bridge? Also, if the stone be projected downwards with a velocity of 3 feet per second, in what time will it strike the water?

3. Two weights sustain each other upon two inclined planes, having a common altitude, by means of a string which is parallel to the planes; find their position, taking into account the weight of the string which is supposed to be of uniform thickness.

4. If a body, whose elasticity is (e), impinge on a plane with a velocity (v), and be reflected with a velocity (v'); and if α, α', be the angles which the directions of its motion, in the two cases, make with a perpendicular to the plane, prove that $\tan.\alpha = e \tan.\alpha'$, $v \sin.\alpha = v' \sin.\alpha'$.

5. The locus of the centres of gravity of the areas of all right-angled triangles on the same hypothenuse ($2a$), is a circle whose radius $= \frac{a}{3}$. The locus of the centres of gravity of their perimeters is a spiral, whose equation is $\rho = a\left(\sin.\theta - \sin.\frac{\pi}{4}\right)$; the pole being in the middle point of the hypothenuse, and θ measured from that line.

6. A hemispherical bowl is terminated by a cylindrical rim, having the same external and internal radius; what must be its breadth, in order that the vessel may rest upon any point of its spherical base?

7. A perfectly elastic ball is projected from the middle point of the base of a vertical square towards one of the angles, and after having been reflected at the sides containing that angle, falls at the opposite angle; find the velocity of projection.

8. Two weights P and Q, connected by a string of given length, are placed upon an inclined plane of the same length, P being at the highest point, and Q at the lowest; after what time will their distance be a minimum?

9. A uniform rod whose length equals twice the diameter, passes through a hole in a spherical shell, and rests with one end against the internal surface; shew that if α be its inclination to the vertical, when in its position of stable equilibrium, $\frac{1}{3}(\pi + \alpha)$ and $\frac{1}{3}(\pi - \alpha)$ will be its inclinations, in its positions of unstable equilibrium.

10. A uniform rod of given length (a) is bent into the form of a cycloid, and oscillates about a horizontal line joining its extremities; prove that the length of the isochronous pendulum $= \frac{a}{5}$.

11. A convoy moving uniformly along a road which runs east and west, is perceived at the instant it is due south of a battery; at what elevation, and towards what point of the compass, must a cannon, loaded with a given charge, be fired at the same instant, so as just to hit it?

12. A beam having one end on a vertical, and the other on a horizontal plane, is kept at rest by a string connecting its centre of gravity with the point of intersection of the planes; find the tension of the string; explain the result when the beam is of uniform thickness.

13. A rod whose length $= a$, and distance of its centre of gravity from one end $= b$, is placed with that end on the arc, and the other on the minor and vertical axis of an ellipse, whose semi-axes are (a) and (b); shew that it will rest in any position; but that if it be disturbed, its centre of gravity will describe the major axis, and its centre of instantaneous rotation, a circle radius $(a - b)$ about the centre of the ellipse; also find the angular velocity of the rod in any position, and the velocity of its centre of gravity.

QUEEN'S COLLEGE, 1826.

1. If two equal forces sustain each other by means of a string passing over a tack, shew that either force : the pressure upon the tack :: 1 : 2 cosine of half the angle at which the forces act.

2. *P* and *W* being in equilibrio on an inclined plane, if the whole be put in motion, then

$$P\text{'s velocity} : W\text{'s velocity} :: W : P.$$

3. *AD* is horizontal, *DC* vertical, *Q* a weight connected with one extremity of a beam *AB* by a string passing over a pulley at *C* in such a manner that *CB* is vertical. Find *Q* when there is an equilibrium, and shew that the equilibrium will be maintained whatever be the position of the beam *AB*, *CB* remaining vertical.

4. A body is suspended from a given point in a horizontal plane, by a string of known length, which is thrust out of its vertical position by a rod (without weight) acting from a given point in the plane, against the body; shew that the tension of the string varies inversely as the tangent of the inclination of the rod to the horizon.

5. A quadrant being placed with one of its sides vertical, and a uniform chain equal in length to the arc being suffered to hang over it from a pin at the upper extremity, it is required to find the sustaining force.

6. *P* and *Q* are two weights connected by a string passing over a fixed pulley, whereof *P* is the greater, at the end of t'' an additional weight (q) is annexed to *Q*. Find the velocity of *P* after any assigned time.

7. If θ be the angular distance of a body from the lowest point in a circular arc; shew that the force in the direction of the arc is to that in the direction of the chord as $2\cos.\frac{\theta}{2} : 1$.

8. Find the centre of gravity of a frustum of a pyramid.

9. An imperfectly elastic ball is projected vertically downwards from a given point with a given velocity, upon a hard horizontal plane; required the sum of the space described by it before the motion ceases, and the whole time of its motion.

10. A ball is projected in a given direction with a given velocity,

determine the time of flight in terms of the oscillations of a pendulum of given length.

11. If a body descends down any curve by the action of gravity, the velocity acquired at any point will be the same as if the body had descended down the same vertical space falling freely.

12. Required the length of a pendulum which vibrates as often in a minute as inches in length.

QUEEN'S COLLEGE, 1827.

1. THE uniform lever AC of given weight and length, and turning on the fulcrum C, has two given weights w_1, w_2 suspended from the extremity A, and the middle point B, and is kept at rest by the given weight P acting at A by means of a string passing over the fixed pulley D, CD is horizontal and equal to CA: find the position of equilibrium.

2. A paraboloid of which the parameter is given, being cut off by a plane perpendicular to its axis, when prevented from sliding, just stands on an inclined plane of given inclination: required the length of the axis of the paraboloid.

3. If the power and weight in equilibrio on an inclined plane be put in motion, the velocity of the power : that of the weight :: the weight : the power.

4. If any number of forces act on a point, the sum of their moments referred to any other point is equal to the moment of their resultant referred to the same point.

5. Suppose (p) to be retained on the curve of a given ellipse the axis major of which is vertical, by a force (q) parallel to the horizon, find the position of equilibrium.

6. Find the centre of gravity of the eighth part of a sphere, *i. e.* of the solid generated by the revolution of a quadrant about one of its sides through 90°.

7. In the direct impact of two inelastic bodies, the difference between the sums of the products of each body into the square of its velocity before and after impact, is equal to the sum of the products of each body into the square of the velocity gained or lost by it.

8. Shew that a body projected in an oblique direction along an inclined plane describes a parabola, and find its latus-rectum, having

given the inclination of the plane and the velocity and direction of projection.

9. Investigate an expression for the time of an oscillation in a cycloid.

10. Shew that the centres of suspension and oscillation are convertible points.

11. Find the time in which a cylinder of uniform density would *roll* down an inclined plane, friction not permitting it to slide.

12. Find the nature of the curve in which a body will descend from one given point to another in the least time possible.

QUEEN'S COLLEGE, MAY 1828.

1. A BENT lever of uniform thickness and density when supported at the angle rests with the shorter arm horizontal. If the shorter arm were twice as long it would rest with the other horizontal. Compare the lengths of the arms, and find the angle at which they are inclined.

2. *AB* is a given horizontal line, *BC* a uniform beam of given weight and length, moving freely in a vertical plane about *B*. A given weight is suspended by a string fixed at *A* and passing through a pulley at *C*. Find the position of equilibrium.

3. In a system where each pulley hangs by a separate string and the strings are inclined to each other at given angles, find the relation between power and weight when there is an equilibrium.

4. A body is placed in the centre of gravity of a triangle which is supported at the three angles. Shew that the pressures at the angles are all equal.

5. Find the centre of gravity of the area which is the difference between similar sectors of two given concentric circles.

6. With what velocity must a cycloid revolve about its axis that the centrifugal force may be sufficient to sustain a particle in the middle of the curve?

7. A ball, the elasticity of which is (e), projected from a given point in the circumference of a circle after being reflected from it twice returns to the given point. Required the direction of projection.

8. Determine that diameter of a circle down the last half of which a body descends in the time down the whole vertical diameter.

9. The thread by which a body oscillating in a given cycloid is suspended, breaks when the body is at the lowest point; after which it proceeds and strikes the vertical tangent to the extremity of the cycloid in the time in which it would have completed its oscillation; required the length of the arc through which it had descended.

10. A circular area the density of which varies as the distance from the centre oscillates in its own plane about a given axis. Required the time.

11. A given weight (p) is suspended at the extremity of a pendulum of given length; where must another weight (q) be fixed, that the whole may vibrate in the least time possible?

12. If a body descends by gravity down the quadrant of a circle touching a horizontal line at its highest point, where will it leave the curve, and where will it cut the horizontal radius produced?

QUEEN'S COLLEGE, MAY 1829.

1. A STRAIGHT lever of uniform thickness, the length and weight of which are given, has two weights P and Q attached to its extremities, and is sustained by a fulcrum at a given point, and also by another fulcrum, on which it presses with a given force; required the position of that other fulcrum.

2. A chord PAQ is knotted to a fixed point A, and drawn in different directions by the forces P and Q, so that the pressure upon $A = \dfrac{P+Q}{2}$; find the angle at which the directions of P and Q are inclined to each other.

3. A weight W is drawn up an inclined plane by a force P, acting by means of a wheel and axle, placed at the top, so that the string attached to the weight may be parallel to the plane. Given R and r the radii of the wheel and axle; find the inclination of the plane.

4. A uniform bent lever ABC, containing a given angle at B,

and having its arms of given weights and lengths, hangs freely by the extremity A: find the position of equilibrium.

5. Find the centre of gravity of a parabolic area terminated by a straight line, inclined at a given angle to the axis.

6. Find the centre of gravity of half the solid intercepted between the surfaces of a hemisphere and paraboloid, on the same base: the latus-rectum of the paraboloid coinciding with the diameter of the hemisphere, and the solid being bisected by a plane, passing through its axis.

7. Find the equation to the catenary between x and y; measuring from the lowest point.

8. W is kept in equilibrio on an inclined plane by P, which acts by means of a single moveable pulley fixed to W, the strings being parallel to each other and to the plane. Prove that when put in motion $P : W ::$ velocity of W : velocity of P, in the directions of their action.

9. Two equal hard balls are projected at the same instant towards each other, from the two extremities of a vertical line, each with the velocity which would be acquired in falling down it. Where will they meet? And what time will intervene between their impact, and their arrival at the lower extremity of the line?

10. An imperfectly elastic ball from a given height is let fall on a given inclined plane; required the point in which, after reflection, it will strike the plane again.

11. The cord which sustains a body in its descent down the arc of a semicycloid breaks at the lowest point. Where will the body strike the tangent at the other extremity of the cycloid? And, if it be perfectly elastic, where, after reflection at that tangent, and at what angle, will it strike the axis of the cycloid produced?

12. A body oscillates in a cycloid; compare the time of descending down one side and ascending half up the other, with the time of describing an arc of equal length cut off by a line parallel to the base.

13. A chain coiled on the ground is drawn up by means of a weight P, connected with it by a string without weight, passing over a fixed pulley; find the velocity of P, after it has descended through a given space.

14. The weights P and Q being collected at the extremities of a uniform rod PQ, of which the length is l, and weight w, the system oscillates about an axis passing through the middle of the rod. Required the time of an oscillation.

15. The radius of the outer surface of a hollow cylinder being given; required its thickness, that the time of its rolling down an inclined plane, may be to the time in which it would descend without friction in the ratio of 5 to 4.

QUEEN'S COLLEGE, MAY 1830.

1. A STRAIGHT lever is sustained on a fulcrum at the middle point, and is kept at rest by two given weights; where must they be placed, that the distance of the one from the fulcrum, may equal the distance of the other from the extremity? And, where must the *fulcrum* be placed, if these situations of the weights be reversed?

2. A weight P is sustained on an inclined plane by three forces, each equal to $\frac{P}{3}$, one acting vertically upwards, another parallel to the plane, and the third parallel to the horizon; required the plane's inclination.

3. A uniform rod of given weight and length, having a weight, which is also given, fixed at one extremity, rests upon a point A, with its other extremity, against a vertical plane BC; find its position.

4. By a straight line drawn through a given point in the diameter of a semicircle, cut off a part, such that what is left may balance, when suspended at the given point with the remainder of the diameter horizontal.

5. A uniform rod of given weight and length suspended at a given point, is drawn out of the vertical position, by a given force, acting from its lower extremity by means of a cord, which passes over a pulley fixed at a given point, in the same horizontal line with the axis; find the angle through which the rod is drawn.

6. Any number of forces being given, acting in any directions upon a rigid body; reduce them to two sets of forces, one set being in a given plane, and the other perpendicular to it.

7. Prove Guldinus's property of the centre of gravity of a revolving plane; and apply it to find the centre of gravity of a semicircle, by the revolution of which about the diameter a sphere is generated; it being granted that the area of semicircle $= \frac{\pi r^2}{2}$, and content of sphere $= \frac{4}{3}\pi r^3$.

8. Prove that if the direction of a body projected from one extremity of the diameter of a circle, along a horizontal plane be such that after one reflection at the curve, it may pass to the other extremity, the sine of the angle which that direction makes with the diameter $= \frac{\sqrt{e}}{\sqrt{(1+e)}}$, e being the elasticity; and find the sine of the angle made with the diameter, by the course of the body after the second reflection.

9. A body descends by gravity, and describes in the nth second of its fall, a space $= p$ times the space described in the last but n; required the whole space.

10. A body of given elasticity is projected upward with a given velocity to strike a horizontal plane, and in t'' returns to the point of projection; required the distance of the plane from that point.

11. A body of given elasticity is projected from the top of a tower in the direction toward a given object on the horizontal plane below, but falling short, it strikes the proposed object after one reflection; required the velocity of projection.

12. Prove that the oscillations of a pendulum through small arcs of a circle are nearly isochronous.

13. A uniform bent rod of given dimensions, suspended by one extremity, is slightly put in motion; find the length of the pendulum which oscillates in the same time.

14. The inscribed equilateral triangular prism, being cut out of a cylinder; find the time in which the remainder will roll down a given inclined plane.

15. At what distance from the centre of a sphere in free space must an impulse be given, that the motion afterwards may be precisely that of a rolling sphere?

16. A given triangle revolves in its own plane, about an axis passing through one of its angles, with a given angular velocity; find the pressure upon the axis.

CORPUS CHRISTI COLLEGE, 1827.

1. If two adjacent sides of a parallelogram represent the magnitude and direction of two forces, prove that the diagonal of the same parallelogram will represent the magnitude and direction of the resultant of the forces: and if two forces be in the ratio of 3 : 2; find their inclination when their resultant is a mean proportional between them.

2. Give a general demonstration of the principle of virtual velocities, and apply that principle to compare the power and weight in case of equilibrium on the screw.

3. Prove that the *power* of any combination of machines is equal to the product of the *powers* of all the simple machines in that combination.

4. Compare the power and weight which keep each other in equilibrium on the system of pulleys where each string is attached to the weight, the number of moveable pulleys being three, and their weights equal; and explain what advantages this system possesses over that in which each pulley hangs by a separate string.

5. If the common balance consist of a given uniform beam AB the centre of gravity of the whole being at G the middle point, but the fulcrum at C a given distance above G; required the position in which it will rest when loaded with unequal weights at A and B.

6. Given the distances of any bodies considered as points from a plane; find the distance of their centre of gravity from the same plane; and find the centre of gravity of a given sector of a circle.

7. Explain the principle on which rocking-stones are made; and if a stone consisting of a given hemisphere, and a cylinder of the same radius be placed on a horizontal plane, find the length of the cylindrical part, so that the stone may be in equilibrium on whatever part of the hemisphere it rests.

8. If x and y be co-ordinates to the catenary measuring from the vertex, prove that $x + a = \frac{a}{2}(e^{\frac{y}{a}} + e^{-\frac{y}{a}})$ where a is the tension at the vertex; and prove that the tension at the point $(x, y) = a + x$.

9. If a roof consist of beams forming an isosceles triangle with its

base horizontal, and a given weight be placed at the vertex, the weights of the beams which are equal to one another being also given; required the horizontal force at the foot of either of the beams. Explain how the effects of this pressure are usually provided against in buildings.

10. In a bridge if the intrados be a circle, and the joints in the direction of radii, to find the extrados.

11. A given beam AB is supported on the horizontal plane DE by means of given weights P and Q hanging freely over pulleys at E and C; find the position of equilibrium of AB.

12. Ten men of equal strength wishing to pull down a tree of given height, and at the same time to avoid all danger from its fall, fix two ropes at its top, one of which reaches to the ground making an angle of 60° with the tree, and the other 45°. Now if four of the men pull at the extremity of the former towards the south, and six at that of the latter towards the south-east: towards what point of the compass will the tree fall?

13. When two imperfectly elastic bodies moving with given velocities and in given directions impinge upon each other; required their velocities and directions after impact, and prove that the motion of their centre of gravity is not affected by the impact.

14. If a body be moved from rest by a constant force f, prove that the space described in time $(t) = \frac{1}{2}ft^2$, and if the body be projected in direction of force with velocity (u), space in time

$$(t) = tu + \tfrac{1}{2}ft^2.$$

15. If a body be projected with a velocity (v) in a direction making an angle (α) with the horizon; find the focus of the parabola described, and prove that its latus rectum $= \dfrac{2v^2 . \cos.^2 . \alpha}{g}$.

16. Prove that the time of oscillation in a cycloid

$$= \pi \sqrt{\frac{L}{g}}, \text{ where } \pi = 3{\cdot}14159\ldots$$

and $L = \frac{1}{2}$ length of the whole cycloid, and g the force of gravity at the place where the oscillation is performed.

17. The force of gravity varying as $\dfrac{1}{\text{dist.}^2}$ from the centre of the Earth, how high must a seconds' pendulum be carried above the

level of the sea that it may vibrate $59\frac{1}{2}$ times in a minute, the radius of the Earth being 3958 miles?

18. A body is projected with a velocity of 160 feet per second, and at an angle of 45° with the horizon, and after a lapse of $5''$ an object dislodged by the projectile strikes the ground; required the distance of the object struck from the point of projection.

19. Explain what is meant by the centre of oscillation; and find the time of a small oscillation of a cone about an axis perpendicular to the axis of the cone, and passing through its vertex.

20. Prove that the times of a solid cylinder and a globe *rolling* down the same inclined plane from rest are in the proportion of

$$\sqrt{15} : \sqrt{14}.$$

21. Explain the construction of the Ballistic pendulum; and if a ball P strike it with a velocity V at a distance (a) from its axis, prove that

$$V = 2 \sin.\frac{\theta}{2} \cdot \frac{P + M}{P} \cdot \frac{h}{a} \sqrt{(g)}l,$$

where $M =$ mass of pendulum, l and h the distances of its centres of oscillation and gravity from the axis, and θ the angle through which it oscillates.

CORPUS CHRISTI COLLEGE, MAY 1828.

1. If two weights balance each other on a straight lever, prove that they are inversely as their distances from the fulcrum, and the converse.

2. If two adjacent sides of a parallelogram represent two forces in magnitude and direction, prove that the diagonal is their resultant: also if a body has two *uniform* motions communicated to it at the same instant along two adjacent sides of a parallelogram, prove that it will move *uniformly* along the diagonal.

3. Compare the power and weight in case of equilibrium on a system of (n) moveable pulleys where each hangs by a separate string, the weights of the pulleys being A_1, $2A_1$, 2^2A_1, &c.... $2^{n-1}A_1$, (A_1 the highest).

4. If any number of forces act in the same plane on a rigid body, find the magnitude of their resultant, and the equation to the line

in which it acts; and prove that the moment of the resultant referred to any point in the plane equals the sum of the moments of the component forces.

5. Define virtual velocity; and in the cases of equilibrium on the bent lever and the screw, prove that $P \times P$'s virtual velocity $+ W \times W$'s virtual velocity $= 0$.

6. Find the centre of gravity of any number of bodies considered as points in space. Find also the centre of gravity of a shell the exterior surface of which is a cone and the interior a paraboloid of half the height of the cone, and the radius of its base two-thirds of that of the cone.

7. Prove that a body placed on a horizontal plane will be supported or not according as a vertical line passing through the centre of gravity falls within or without the base; and find where a right-angled isosceles prism must be cut by a plane parallel to one of its equal sides that it may just be supported when placed with this section on a horizontal plane.

8. Investigate Guldinus' Theorem for finding the content of a solid of revolution: and if a right-angled triangle, the sides of which are (a) and (b), generate a solid of revolution about its hypothenuse, prove by this theorem that its content $= \dfrac{\pi a^2 b^2}{3\sqrt{(a^2+b^2)}}$.

9. If a chain of given length hangs freely over two given points in the same horizontal line, find the position of equilibrium.

10. If two chords of a circle drawn from the same point represent two forces, and if one of the chords subtends an arc of 90°, find the other when the resultant of the two is the greatest possible.

11. Explain what you understand by *moving force* and what by *accelerating force*, and if a space (s) be described from rest by a constant force f in time (t), prove that $s = \frac{1}{2} ft^2$; and that the spaces described in the 1st, 2nd, 3rd, &c. seconds of time are as the odd numbers, 1, 3, 5, &c.

12. Two bodies A and $2A$ whose elasticity : perfect elasticity :: 2 : 3, move with velocities $2v$ and v respectively, and the direction of each makes an angle of 30° with the common tangent at the point of impact; find the directions and velocities of the bodies after impact.

13. If a body be projected in vacuo, find the equation to its path, and shew that it is a parabola; and the velocity and direction of projection being given, find the range on a given inclined plane, and the time of flight.

14. If a body oscillates in a cycloid, prove that the accelerating force along the curve is proportional to the arc intercepted between the body and the lowest point; and find the time of oscillation: and if the body oscillates through the whole arc of the cycloid, and the time of oscillation be divided into four equal intervals, compare the arcs passed over in the 1st and 2nd of them.

15. Define the *centre of gyration*, and find it in a sphere of uniform density revolving about a diameter.

16. Define the *centre of oscillation*, and find it in the solid formed by the revolution of a sector of 60° of a given circle about one of its extreme radii; the solid being suspended from the vertex.

17. If bodies be projected from the same point, and with the same velocity, but in all possible directions in the same plane; find the locus of the points at which the path of each body is inclined at 45° to the horizon.

18. If two equal cylindrical wheels be *fixed* on a given cylindrical axle, and the whole be caused to roll along a horizontal plane by a weight unwinding a string from the axle, find the velocity along the plane at any time, and the space described: and find the magnitude of the obstacle which placed under either wheel will just prevent its motion. If the plane and wheel be perfectly smooth what will be the nature of the motion?

CORPUS CHRISTI COLLEGE, JUNE 1829.

1. IF any number of forces, $PQRST$, &c., $P'Q'R'S'T'$, &c. acting at the arms of a bent lever to turn it different ways, balance one another, and $pqrst$, &c., $p'q'r's't'$, &c. be perpendiculars on their directions from the fulcrum, then shall

$$Pp + Qq + Rr + \&c. = P'p' + Q'q' + R'r' + \&c.$$

2. If any two forces acting at the same point be represented in magnitude and direction by the sides of a parallelogram, the diagonal also represents their resultant in magnitude and direction.

3. In the screw, $P : W ::$ distance between two threads : circumference of the circle described by the power.

4. Prove the principle of virtual velocities in the wedge generally.

5. If a system be in equilibrium, the centre of gravity is at its highest or lowest point; required a proof.

6. If h k be the co-ordinates of the centre of gravity of a curvilinear area, shew that

$$h = \frac{\int x dm}{\int dm}, \qquad k = \frac{\int y dm}{\int dm}.$$

Apply the expressions to a quadrant of an ellipse.

7. In an arch which is in equilibrium, the weights of the voussoirs are as the differences of the tangents of the angles which their joints make with the vertical.

8. A solid is formed by the revolution of a semi-parabola through an angle of 60°. Find the centre of gravity.

9. Find the resultant of any number of forces acting in the same plane on a rigid body, and determine also the equation to the straight line in which it acts.

10. A cone and hemisphere of equal bases and altitudes are placed on an horizontal plane, the extremities of their bases coinciding; find the position of equilibrium of a given rod (in length less than the diameter of the base) which shall rest between them.

11. Find the equation to the catenary between x and y beginning from the lowest point.

12. Explain and prove the first law of motion.

13. Two bodies, whose common elasticity is e, and moving with given velocities, impinge directly on each other; determine their velocities after impact.

14. Prove the expressions,

$$s = \tfrac{1}{2}ft^2, \qquad f = \frac{v^2}{2s},$$

and shew that the spaces described in the 1st, 2nd, 3rd, 4th, &c. seconds are as the numbers, 1, 3, 5, 7, &c.

15. Shew that the curve described by a projectile is a parabola, and find its latus rectum.

16. Two spherical bodies move uniformly in any two straight lines not in the same plane, if they meet, determine their concourse, if not, find when they approach nearest to one another.

17. A body perfectly elastic is projected up a plane whose angle of inclination equal β, at an angle α, and with a given velocity U, after reflexion it ascends vertically upwards, shew that its range on the inclined plane $= \dfrac{U^2 \cos.^2 \alpha}{g \sin.\beta \,.\, \cos.^2 \beta}$, and that the time of flight from the point of projection till the body returns to it $= \dfrac{6 \,.\, U \,.\, \cos.\alpha}{g \,.\, \sin.2\beta}$.

18. A body descends down an inverted cycloid; find the time of a whole oscillation.

19. Find the equations of motion of a body moving in a plane and acted on by any forces in that plane.

20. A body is acted on by a force tending to a centre; find the polar equation to the curve described.

21. P draws up W by means of the wheel and axle; find the accelerating force on P.

22. The strength of a string by which a body may be whirled round in a vertical circle, must be able to support six times the weight of the body.

23. A paraboloid resting on an horizontal plane has its axis drawn a little from the vertical, and begins to oscillate; find the time of one of its small oscillations.

24. Define the centre of oscillation, and shew that the centres of oscillation and suspension are reciprocal.

CAIUS COLLEGE, MAY 1830.

1. If two weights balance each other on a straight lever, their distances from the fulcrum are inversely as their weights.

2. If any two forces act at the same point, they are equivalent to a single force whose magnitude is expressed by the diagonal of the parallelogram, of which the sides express the magnitude and direction of the two forces.

3. If any number of parallel forces P, P', P'', &c. act on a system x, y, z, x', y', z', &c. being the co-ordinates of their points of application, and if x, y, z be the co-ordinates of the centre of forces; shew that

$$x = \frac{P.x + P'x' + P''x'' + \&c.}{P + P' + P'' + \&c.},$$

$$y = \frac{P.y + P'y' + P''y'' + \&c.}{P + P' + P'' + \&c.},$$

$$z = \frac{P.z + P'z' + P''z'' + \&c.}{P + P' + P'' + \&c.}.$$

4. Find the centre of gravity of a pyramid whose base is a triangle.

5. Prove Guldin's theorems.

6. Given the resultant R of two forces P, Q, their sum, and their inclination; to find the forces.

7. P supports Q on a given inclined plane, P's direction making with the plane an angle ϕ; to find the conditions of equilibrium.

8. Determine the same, taking into account friction.

9. In a system of pullies, each string is attached to the weight. Determine the equation of equilibrium, taking into account the weight of the blocks.

10. Find the equations to the common catenary, and if $t =$ the tension at any point, $\rho =$ the radius of curvature, and $c =$ tension at lowest point, then $\rho = \frac{t^2}{c}$.

11. A beam (PQ) rests upon two inclined planes; to find the position of equilibrium.

12. Prove the principle of virtual velocities for the five mechanical powers.

13. Two bodies m, m' move in the same direction with velocities v, v', and v is greater than v', determine their motions after impact, first when perfectly hard, and next when their common elasticity is ϵ.

14. When a body is acted upon by a uniformly accelerating force shew that

$$s = \tfrac{1}{2} gt^2, \qquad v = gt, \qquad \text{and } s = \frac{v^2}{2g}.$$

15. A body is projected from a given point in a given direction with a given velocity; prove that

$$y = x \,.\, \tan.\alpha - \frac{gx^2}{2 \,.\, a^2 \cos.^2 \alpha}$$

is to the equation to the curve, and shew the curve to be the common parabola of which the parameter $= \dfrac{2 \,.\, a^2 \cos.^2 \alpha}{g}$.

16. The velocity acquired in falling down any curve = the velocity acquired by falling freely down the same vertical height.

17. Find the time of oscillation in a common cycloid, and in a circle.

HYDROSTATICS.

TRINITY COLLEGE, 1822.

1. When a fluid is at rest its surface is horizontal.

2. Shew how the capacity of an irregular vessel may practically be found with accuracy.

3. Between every two positions of stable equilibrium of a floating body there is a position of unstable equilibrium.

4. Supposing that it requires one nth part the weight of a particle of water to overcome its adhesion: find what is the least slope down which water would flow.

5. It is found that on mixing 63 pints of sulphuric acid at 1·82 specific gravity, with 24 pints of water, one pint is lost by their mutual penetration: find the specific gravity of the compound.

6. Find the whole pressure on the side of a hollow octahedron filled with water,

(1). When suspended by an angle.

(2). When by two adjacent angles, with the intermediate edge horizontal.

(3). When by the three angles of a side, so that this side is horizontal;

the length of an edge being a inches: and the weight of a solid foot of water being 1000 ounces.

7. How far may a shaft AX be bored perpendicularly into the vertical face of a canal-bank before the bank would be blown up; supposing that the tenacity of the earth would be accounted for by an addition to its specific gravity, and that the form of the bank is determined by the equation $PM = f(AM)$, (AM) being the vertical line from the point P on the bank, to M in the shaft.

Apply the result to the case where AB is 7 feet, BC is 2 feet, the bank is a plane of 30° elevation; the specific gravity of the earth being 2·25, and the addition for adhesion being $\frac{1}{5}$ of the weight.

8. A hollow sphere filled with water is divided by a vertical section through the centre; find how long the water will be in escaping if the two halves are separated by a given small interval from each other.

9. A cylinder filled with water empties itself through a hole in the side, whose area is a^2, at the height b. After the times r and s it is observed to spout to the distance m and n respectively from the foot of the cylinder. Find the dimensions of the cylinder.

10. A piston of given area a^2 working in a cylinder, forces water through a pipe of given aperture c^2 opening upwards. Find the rate at which the piston must move, and the force that must be applied to it, to make the water spout to the height h.

11. A fluid spouts in a vertical direction; find the form it should assume.

12. Explain the phenomenon of capillary attraction; and deduce the relation between the diameter of the bore, and the height at which it will sustain a given fluid.

13. Investigate the density of the air at a given height above the earth's surface; and shew how the result may be applied to the ascertaining of the height of mountains.

14. A bucket of water is whirled round in an horizontal circle: find the form which its surface will assume when the suspending string is inclined at an angle of 60° to the horizon.

15. Find the velocity of a ball in an air-gun, supposing the resistance of the compressed air before the ball to vary as the velocity.

16. What is the nature of the curve of isochronous oscillations when the resistance is as the square of the velocity.

TRINITY COLLEGE, 1823.

1. A Spring on the side of a hill appears on the same level with the foundation of a house on an opposite hill, known to be at the distance of a mile; within what distance of the true level of the house can water be conveyed in pipes from the spring?

2. A circle is just immersed in a vessel of fluid; divide it by another circle touching it internally at the surface into two parts, such that the pressures upon them shall be as $n : 1$.

3. If the specific gravities of fluids be proportional to the parts of the axis of an hydrometer extant above the fluid; what is the form of the stem, considered as a solid of revolution?

4. Supposing the specific gravity of a man, of water, and cork, to be 1120, 1000, and 240 respectively; what quantity of cork must be connected to a man weighing 150lbs., that he may just float in the water?

5. Illustrate the nature of stable and unstable equilibrium, by the instance of a solid elliptic cylinder placed in a fluid of twice its specific gravity.

6. If a barometer, having its lower end immersed in a bason of mercury, be suspended from the beam of a balance, it is found to weigh as much as when it is inverted, filled to the same height, and suspended by the other end. Explain this circumstance.

7. If B equal the altitude of the barometer at a lower station and b at a higher, and A be the modulus of the atmospheric system of logarithms, prove that the altitude (a) may be found without logarithms very nearly by this formula:

$$a = \frac{1}{2}\frac{AB}{b} - \frac{1}{2}\frac{Ab}{B}.$$

8. Abstracting from gravity, suppose that any number of forces are in equilibrium when applied to the pistons of cylinders, which are fitted to apertures in a vessel of water; prove that the general principle of virtual velocities obtains in a machine of this nature.

9. The velocity with which an elastic fluid rushes into a vacuum is independent of its density.

10. Having given the velocity of a vessel, and also that of the wind, and the angle made by the wind and keel; find the angle made by the keel and sail when the propelling force of the wind is the greatest possible.

11. If steam from a boiler, the pressure of which on the unit of surface is P, be admitted above the piston of a cylinder, (diam. $2r$) while it descends through a given space s, and then, the communication being closed, the steam be left to expand; prove that the

whole effect (estimated on the principle of the vis viva) of the steam acting through the additional space S, will be

$$E = P\pi r^2 s\left(1 + \log.\frac{S}{s}\right).$$

12. If the resistance vary as the velocity, and gravity act uniformly in lines perpendicular to the horizon, the velocity of the projectile at any point is as the tangent produced to the asymptote. [*Newton*, Vol. II. Prop. 4.]

TRINITY COLLEGE, 1824.

1. DEFINE a fluid: and an elastic fluid.

2. Explain the action of a syphon.

3. The whole pressure on any surface immersed in a fluid, is equal to the weight of a column of the fluid, whose base is equal to the surface pressed, and its height equal to the depth of the centre of gravity of that surface below the surface of the fluid.

4. Investigate the rule for finding the centre of pressure, and apply it to the case of a triangle, a side of which is coincident with the surface of the fluid.

5. A hemispherical cup is filled by a solid hemisphere, which turns round a fixed horizontal diameter as an axis: shew, that if the solid be turned about, and a fluid of double the specific gravity of the solid be poured into the cup, then, upon the solid being allowed gently to return, the fluid, whatever was its quantity, will rise exactly to the edge of the cup, and none will escape. (The principle of Mr. Cecil's Lamp.)

6. Required the weight of a hydrometer, which sinks as deep in rectified spirits (specific gravity ·866), as it sinks in water, when it is loaded with 67 grains.

7. A balloon of given weight and capacity is so constructed, that, as it rises, the air escapes, till the elasticities of the internal and of the external air are equal. Compare the greatest it can attain, with that which it could have attained, if the air had not been suffered to escape.

8. If a barometer be placed under the receiver of an air-pump, the sum of the altitudes in the barometer and in the gage will be constant.

9. Explain the construction of a force pump; and, supposing the water to be forced in given quantities, and at given intervals, into an air-vessel of known capacity, find the greatest and least velocities of the issuing stream.

10. Given the breadth of a wave, and the rate of its apparent motion, to find its height.

11. Given the situations of a bucket of an overshot wheel where, independently of the centrifugal force, the water would begin to leave it, and where it would be wholly discharged; find, either by a geometrical construction, or algebraically, the corresponding situations when the centrifugal force is considered.

12. Compare the resistances to the motion of a cube in a fluid, when it moves in the direction of its longest diagonal, and when it moves in the direction of an edge.

13. Investigate the relation between the velocity of a machine driven by the direct impulse of water, and that of the water, that the effect may be a maximum.

14. Required the greatest velocity of a heavy sphere sinking in a fluid; and apply the result to determining the specific gravity and magnitude of metallic spherules, too minute to be measured or weighed.

15. How must the density of a resisting medium vary, that a heavy body, projected horizontally, may describe the quadrant of a circle?

TRINITY COLLEGE, 1827.

1. A GIVEN quantity of fluid is contained in a bottle of given base. If by narrowing the upper part of the bottle the fluid be made to stand higher, how will the *pressure* on the base be affected, and how the weight of the vessel?

2. If a body float on a fluid, the part immersed : whole body :: specific gravity of body : specific gravity of fluid.

3. Two bodies of equal weights when connected will just float; what is the relation of their specific gravities and of that of the fluid?

4. From a thin hollow iron sphere of which the radius is a, a segment, of which the abscissa is b, being cut, will just float; what is the whole weight of the sphere?

5. Define a *level surface;* and shew that the pressure at all points of a level surface, in any fluid in equilibrium, is the same.

6. Prove the expressions for the co-ordinates of the centre of pressure,

$$X = \frac{\int\int x^2 dx dy}{\int\int x dx dy}, \quad Y = \frac{\int\int xy dx dy}{\int\int x dx dy},$$

and apply them to the case of a quadrant with its radii horizontal and vertical, and its convexity downwards.

7. In the oblique position of flotation of an isosceles triangle, the circle described through the vertex and the extremities of the line of flotation will bisect the base.

8. In a level it is found that the bubble moves $\frac{1}{10}$th of an inch for each change of 1″ in the inclination of the tube. What is the form of the interior upper surface, and its curvature?

9. The Thames tunnel consisted of a horizontal way 30 feet wide, 15 feet high, and 580 feet long, and of a circular vertical shaft 50 feet diameter; supposing the river to be 30 feet deep, and to rush in through an aperture of 1 foot square;

(1). How long would the horizontal way take in filling?

(2). How long would the shaft take in filling?

10. The transverse section of the immersed part of a boat being a semicircle; find the direction and magnitude of the whole pressure on one side of the section.

11. Find the resistance upon the prow of a boat supposed to be in the form of a quarter of a sphere bounded by a horizontal plane, and a plane perpendicular to the direction of its motion.

12. In a capillary tube, what part of the tube is employed in sustaining the fluid, and how is this proved? Shew that the height at which the fluid will stand will be inversely proportional to the diameter of the tube.

13. In an imperfect barometer tube, of which the length is 33 inches, the mercury stands at 29, when in a perfect one it is at 30: at what height will it stand in the imperfect one, when it is at 20 in the perfect barometer?

14. A rectangular parallelopiped of which the specific gravity is one half, floats with a side horizontal; what is its stability? In what case will the *equilibrium of indifference* subsist?

15. A heavy piston descends by its weight in a tube filled with air; find its velocity at any point, and the depth to which it will descend (neglecting friction.)

16. Explain the common pump. What are the conditions under which it will not work when the piston does not descend to the fixed valve?

17. If the elasticity of the air, including the effect of temperature, vary as the $(\text{density})^{1+m}$, shew that the pressure (p) at any altitude z, will be given by the formula

$$p = gAD\left\{1 - \frac{z}{(m+1)A\left(1+\frac{z}{r}\right)}\right\}^{m+1},$$

A being the height of a homogeneous atmosphere, of density D, and r the radius of the Earth.

Also m being $\frac{1}{3}$; find the whole height of the atmosphere.

TRINITY COLLEGE, MAY 1828.

1. If two fluids communicate in a bent tube, their perpendicular altitudes above the plane where they meet are inversely as their specific gravities.

2. To find the specific gravity of a body lighter than the fluid in which it is weighed.

3. A hollow iron sphere just floats in water: find its thickness. If the exterior and interior surfaces are not concentric, how may the fact be ascertained?

4. Compare *geometrically* the resistance on a semicircle moving

in a fluid perpendicularly to its diameter, with the resistance on its diameter.

5. A close paraboloid containing common atmospheric air is let down in water to a certain depth, and a small orifice being opened at its vertex, which is downwards, the water rises up to the middle of its axis. Determine the depth, and the velocity with which the water first rushed in.

6. A cylindrical bucket, containing a given quantity of water, is whirled round its axis with a given angular velocity, not great enough to throw any water over the top. A small orifice being opened at the centre of the base, find the quantity of water which can escape by it, and the time in which a given quantity will flow out.

7. A sphere full of water is placed on a horizontal plane. Find the point at which a hole being bored through it perpendicularly to its surface, the water shall spout to the greatest distance on the horizontal plane.

8. Determine the position of equilibrium of a homogeneous semi-parabola in a fluid, supposing the axis and the extreme ordinate wholly extant.

9. A vessel full of water has a side loose, whose shape is a given triangle, with the base horizontal, and vertex downwards. Find the magnitude and point of application of a force which shall keep it in its place.

10. Define the *Metacentre*, and show how it may be found. Explain how its position determines the stability of a floating body.

11. The height of a homogeneous atmosphere is the same for all distances above the Earth's surface.

12. Describe the construction and action of the common air-pump, and determine the quantity of air remaining in the receiver after t turns.

13. Determine the curve traced out by the extremity of the horizontal diameter of a small spherical air-bubble ascending in a fluid.

14. Describe the air-gun, and having given the quantity of air and the space occupied by it at first, find the velocity of the bullet at any given point of the tube.

15. The force varies as the distance, and the resistance of the medium as the square of the velocity. Find the velocity of a body descending towards the centre, and shew from your result that when the resistance vanishes, the velocity coincides with that in a non-resisting medium.

TRINITY COLLEGE, JUNE 1829.

1. Find the equation to the surface of revolution, such that when filled with fluid, and placed with its vertex upwards, the pressure on any horizontal section equals the pressure on the curve surface above it.

2. If two fluids communicate in a bent tube, their perpendicular altitudes above the plane where they meet are inversely as their specific gravities.

3. If a lighter fluid rest upon a heavier, and their specific gravities be as $a : b$, and a body, whose specific gravity is c, rest with one part P in the upper fluid, and the other part Q in the lower,

$$\text{then} \quad P : Q :: b - c : c - a.$$

4. Define the *centre of pressure*, and shew how its position may be determined.

5. In the side of a vertical flood-gate is a rectangular aperture, whose sides are vertical. To this aperture is fitted a door moveable, in a direction perpendicular to its plane, about a horizontal axis passing through its centre of gravity. Given the height at which the water stands on one side of the gate; find the height at which it must stand on the other, in order that the door may remain at rest. If the water rise on the deeper side, which way will the door be pushed open?

6. A hemisphere, with a flat lid, filled with fluid, is held with a point in its edge uppermost. Find its position when the sum of the pressures on the curve and plane surfaces is a maximum.

7. It is found that when a small horizontal orifice is opened in the side of a cone filled with fluid at a height $= \frac{1}{4}$ of the cone's height, the fluid spouts to the greatest distance on the horizontal

plane on which the cone is standing. Compare the height of the cone with the diameter of its base.

8. Determine the greatest angular velocity, with which a sphere filled with water may be whirled round its vertical axis, so as to allow the whole of the water to escape by a small orifice at the lowest point.

9. A cone full of water is pierced with innumerable holes perpendicular to its surface. Find the boundary of the issuing fluid.

10. A cylinder whose height is a, and base b, open at the top, and filled with fluid, is placed by the side of a paraboloid of equal height and base, also open at the top. A communication being opened between the vessels by means of a small orifice whose area is $A\varpi^2$, shew that the time which elapses before the fluid stands at the same height in each

$$= \frac{b^2\sqrt{a}}{A^2\sqrt{g}}\{\sqrt{2} - \text{h.l.}\,(1 + \sqrt{2})\}.$$

11. Over two pullies in the same horizontal line passes a string of inconsiderable weight, to which are attached two cylindrical vessels, equal in every respect, containing unequal quantities of water. A small orifice of the same size being simultaneously opened in the base of each, and the vessels being at the same moment allowed to move, find (1) the quantity of water remaining in one vessel when the other is empty, and (2) the time in which this remaining quantity will run out. (N. B. It is supposed that the ascending vessel does not reach the pully towards which it is moving before both are emptied.)

12. Two equal cylinders being placed as in the last problem, but containing equal weights of fluids of different specific gravities; state the effects which will result from simultaneously opening a small orifice of the same size in the base of each.

13. Define the Metacentre, and shew how its position determines the stability of a floating body.

14. In an imperfectly exhausted barometer, the depression below the standard altitude is to the standard altitude as the space which the air left in the tube occupied before immersion is to the space which it occupies after.

15. Describe the construction and action of the common Pump.

TRINITY COLLEGE, MAY 1830.

1. THE equality of the pressure of fluid in all directions may be deduced from the principle, that when a mass of fluid is in equilibrium, the state of rest is not altered by supposing any portion of the mass to become solid.

2. The pressure at any point of a fluid mass of uniform density contained in any vessel and acted upon by gravity, is proportional to the perpendicular depth of the point below the surface of the fluid. In what manner is the pressure estimated, when it is said to be equal to $g\rho h$, g being the measure of the accelerative force of gravity, h the depth below the surface, and ρ the density?

3. The pressure of a fluid against any surface in a direction *perpendicular* to it, varies as the area of the surface multiplied into the depth of its centre of gravity below the surface of the fluid.

4. The pressure of a fluid *downwards* against the sides and bottom of any vessel whatever, is the weight of the fluid contained in it.

5. The weight of P in water was 10 grains, of Q in air 14 grains, of P and Q connected together, the weight in water was 7 grains; the specific gravity of water being 1, and of air ·0013, shew that the specific gravity of Q was ·8237, and that it was as large as 17·023 grains of water.

6. A tube, the upper part of which was bent like a retort, and contained a quantity of powder, was plunged vertically into mercury to a certain depth, after having been partially exhausted of air; the mercury stood in it at the height l, and after the tube was depressed through an additional space a, at the height $l-f$: the powder having been taken out, the tube was exhausted again so that when plunged to the same depths as before, the mercury stood at the heights l and $l-g$; hence shew that if $k^2 =$ the transverse section of the tube, and $h =$ the height of the mercury barometer, the solid content of the powder was

$$-k^2(g-f)\left\{1+\frac{a(h-l)}{fg}\right\}.$$

7. Obtain the general equation of the equilibrium of fluids,

$$dp = \rho (Xdx + Ydy + Zdz);$$

and the equation of condition that must be satisfied that the equilibrium may be possible.

8. When any fluid mass is in equilibrium, the points of equal density lie in continuous surfaces, which are also surfaces of equal pressure.

9. Prove that in the earth's atmosphere, considered independently of the diurnal rotation, the conditions of equilibrium are not satisfied, by reason of the irregular distribution of the heat from the sun, and in consequence that winds continually prevail.

10. A cylindrical vessel of given dimensions containing air, revolves about its axis with a given angular velocity; it is required to find the pressure at the surface of the cylinder, and the law of the variation of the pressure from the axis to the surface.

11. Obtain Laplace's formula for the barometric measurement of heights.

12. If fluid of any kind be moving in such a manner that at the same point in space the velocity is constantly the same in quantity and direction, then if X, Y, Z be the forces impressed at any point whose co-ordinates are x, y, z, and v be the velocity at this point,

$$\int \frac{dp}{\rho} = \int (Xdx + Ydy + Zdz) - \frac{v^2}{2} + C.$$

13. Ascertain the velocity with which air in a large vessel, maintained in a given state of compression, will issue through a small orifice into the atmosphere.

14. Describe the action of the Hydraulic Ram.

15. When a body acted upon by a force tending to a centre, describes a curve in a resisting medium, the velocity at any point is that acquired by falling down one fourth the chord of curvature by the action of the force at that point.

16. If an indefinitely slender column of air, in which

$$p = a^2 D (1 + s),$$

be any how put in motion, and the motion be small, the velocity v

and condensation s, at a distance x from a fixed point, and at a time t reckoned from a given instant, will be given by the equations,

$$v = F(x - at) - f(x + at),$$
$$as = F(x - at) + f(x + at).$$

ST. JOHN'S COLLEGE, DEC. 1820.

1. EXPLAIN the construction and use of a common barometer and its vernier. Construct also a self-registering thermometer.

2. Shew how the specific gravities of solid and gaseous bodies may be determined.

3. A body when put under the receiver of a given air-pump weighs (a) ounces, and after (n) turns weighs (b) ounces. Required the weight of the body in vacuo; and supposing the specific gravity of the body known, determine the density of the air in the receiver at first.

4. A tretrahedron is filled with water. Having given the length of one of its edges, and the pressure on the base, find the pressure on the sides.

5. A given cone filled with water is supported with its axis inclined to the horizon at a given angle. Find on what section parallel to the base, the pressure is a maximum.

6. Given the quantity of air left in a barometer tube before immersion, find the height at which the mercury is supported after immersion. What would be the height if the particles repelled each other with forces varying as $\frac{1}{\text{dist.}^2}$?

7. A diving bell of given dimensions in the form of a hemispheroid, the sections parallel to the base being ellipses, sinks till the water reaches the middle point of its axis; find its depth below the surface.

8. A given frustum of a paraboloid whose density is (d) sinks to a depth (a) in a fluid. How far will a similar and equal frustum, similarly immersed, whose density varies inversely as the distance from the vertex, and at the smaller end is equal to d, sink in the same fluid?

9. The groin $AMPQ$ is generated by the motion of a variable parabola apm parallel to itself, the section through the axes of the parabolas being a right-angled isosceles triangle ANM. Given the dimensions of the upper surface PMQ, find the time of emptying through a small given orifice at A, the vessel being full, and the axis AN vertical.

10. A given cylinder rests in water with its axis vertical, and two-thirds of it immersed. Suppose half the part extant to be suddenly taken off, find the time of an oscillation.

11. A prismatic vessel filled to a given altitude, has a small given orifice made in the middle of its side. Compare the forces at first, and after n'', with which the spouting fluid strikes a vertical plane. The effects being estimated in a direction perpendicular to the plane, whose distance from the vessel is given.

12. Two equal cylinders A, B, whose density is $(2d)$ and altitude (a) are immersed in different fluids, viz. A in a fluid whose density varies as the depth, and B in a fluid of uniform density (d). When connected by a string passing over a fixed pulley they balance in a given position. Supposing B depressed through a space (c), find by how much it must be lengthened to restore the equilibrium.

ST. JOHN'S COLLEGE, DEC. 1827.

1. THE pressure on any surface immersed in a fluid, is equal to the weight of a column of fluid, whose base equals the surface pressed, and altitude the perpendicular depth of its centre of gravity.

2. A body is floating between two known fluids, and the part immersed in the lower is observed to be the same, as if it were floating on the surface of a fluid, formed by the mixture of equal quantities of the two fluids; required the specific gravity of the solid.

3. A sphere is full of fluid; draw that horizontal section which shall sustain the greatest pressure, and compare that pressure with the pressures on the two surfaces into which the sphere is thus divided.

4. Explain the cause, and action of reciprocating springs.

5. A given cylinder is excavated beneath into the form of an hemispheroid, having the same base and altitude. How high may this be filled with fluid, whose specific gravity equals n times that of the cylinder, before it begins to raise it from the horizontal plane?

6. A cone with its vertex upwards is filled with fluid, whose density varies as the nth power of the perpendicular depth, required to compare the pressure on the conical surface with the weight of the fluid.

7. Find the resistance on an oblate spheroid moving in a fluid in the direction of its axis.

8. To find the effect on the graduation of a barometer tube, when the area of the basin is taken into account.

9. A vessel of fluid in the form of a paraboloid is placed with its vertex downwards, find where a small given orifice must be made, that the fluid may continue to issue for the greatest length of time, and compare that time with the time of emptying the whole vessel through the same orifice in the lowest point.

10. The barrel of an air-pump discharges at every stroke into the receiver of a condenser. Required the density in the condenser after n strokes, both vessels being filled with common air at first. Also determine the limit to the increase of density in the condenser.

11. There is a flood-gate communicating with a reservoir of water, in the form of a rectangle, moveable above its upper edge, which is horizontal, and at a given perpendicular depth below the surface of the fluid. Given the weight and dimensions of the flood-gate; required at what angle it must be inclined to the horizon, that it may confine the water by its weight alone.

12. A cylindrical vessel of air revolves above its axis, so that the velocity of its circumference equals that acquired down n times the height of an homogeneous atmosphere. Required the density of the air in the cylinder at different distances from the centre, neglecting the effect of gravity on the air. And shew that when n is very small, the distance of the point where the density remains the same, as when the cylinder was at rest, equals $\frac{1}{\sqrt{2}}$. radius ultimately.

St. JOHN'S COLLEGE, Dec. 1828.

1. Define W, M, and S, and prove that $W = MS$. The specific gravities of pure gold and copper are 19·3 and 8·62. Required the specific gravity of standard gold, which is an alloy of eleven parts pure gold and one part copper.

2. A sphere floats between two fluids of known specific gravity; given the ratio of the portions of surface immersed in each fluid, find the specific gravity of the sphere.

3. A tube of given length is inserted in the side of a vessel of fluid at a given depth from the surface. What must be the inclination of the tube that the latus-rectum of the parabola described by the issuing fluid may be a maximum?

4. If the particles of air repel each other with a force varying inversely as the nth power of the distance of their centres; r = radius of the Earth, h and H the altitudes of a homogeneous atmosphere at the Earth's surface, and the distance x above it, shew that

$$H = h - \frac{n-1}{n+2} \cdot \frac{rx}{r+x}.$$

5. Construct and explain the action of the air-pump.

6. Explain how a rudder acts upon a boat. Give its greatest effect in altering the course; how much is the boat then resisted by it?

7. A body is projected against a stream whose resistance $\propto v^n$, with a velocity equal to that of the stream. Required the space described before the body begins to return; and shew that, when $n = 2$, this space is the same for streams flowing with different velocities.

8. Determine the length of a gun, that the velocity of the discharge may be a maximum, supposing the friction of the barrel to be constant.

9. The vertex of a conical body whose vertical angle is inconsiderable is fixed in a fluid at rest at a depth below the surface less than the length of the cone, and in a known ratio to it. Given the relative specific gravities of the solid and fluid, required the position of equilibrium?

10. The area included between the ordinate and abscissa of a parabolic curve of the nth order is moved into a fluid in the direction of its axis with the ordinates parallel to the horizon. Shew that the path described by the centre of pressure of the part immersed is another parabolic curve of the nth order.

11. A hollow paraboloid rests on a fluid with its axis vertical. A small orifice being made in the lowest point, required the time before the fluid begins to flow in at the top. The orifice is so small the momentum of the descending paraboloid need not be taken into account.

12. The surface of a hollow cone is covered with an infinite number of equal circular apertures in contact with each other. Find the time of emptying when filled with fluid.

ST. JOHN'S COLLEGE, MAY 1829.

1. IF any number of pistons be applied to orifices of different magnitudes in the sides of a close vessel filled with fluid, the force acting upon them to maintain equilibrium will be proportional to the areas of the respective orifices. How will this result be modified when gravity is taken into the account?

2. Describe the hydrostatic bellows; and shew that if there be an equilibrium, and more fluid be poured in, it will rise equally in the pipe and the other part of the machine.

3. When a body is sustained between two fluids, to compare the parts immersed in each.

4. Explain the construction of Nicholson's Hydrometer, and its use in finding the specific gravities of fluids, and small solids.

5. Find the range of a fluid, spouting through a very small orifice in the side of a vertical cylinder standing upon a horizontal plane.

6. If a stream impel a plane opposed to it perpendicularly, the whole force against the plane $\propto Asv^2$.

7. If the distances above the surface of the Earth increase in arithmetic progression, the corresponding densities of the air will decrease in geometrical progression; what supposition is here made respecting the force of gravity?

8. Explain the action of the forcing pump, and determine the pressure on the piston in any position.

9. If X, Y, Z, be the components of the forces which solicit a particle situated any where within a fluid mass, p be the pressure, and ρ the density at the same point, prove that

$$dp = \rho\,(Xdx + Ydy + Zdz);$$

and hence deduce the equations of equilibrium of elastic fluids.

10. Explain the construction of the common barometer; and find the corrections to be applied for the sinking of the mercury in the basin, and for the change of temperature.

11. When a body is sustained in a fluid, and the equilibrium is slightly disturbed, find the effort of the fluid to restore the body to its position of equilibrium, or to carry it further from it; and in the former case, determine the times of the small oscillations which ensue.

12. Find the time of emptying a vessel through a small orifice; and shew that in the case of a vertical prism, the descending surface is uniformly retarded.

ST. JOHN'S COLLEGE, MAY 1830.

1. If there be two or more fluids of different densities at rest in the same vessel, the common surface of every two is horizontal.

2. The sum of the perpendicular pressures on any surface immersed in a fluid, is equal to the weight of a column of fluid whose base is the area of the surface, and height the depth of its centre of gravity.

3. The pressure against the sides of a vessel containing fluid, when estimated in the direction of gravity, is equal to the weight of the fluid.

4. Find the specific gravity of a fluid by the common hydrometer.

5. Find the altitude of the mercury in the barometer, when a given quantity of air has been allowed to remain in the tube; explain the meaning of the two roots.

6. Shew that as the distance from the Earth's surface increases, the density of the atmosphere diminishes; find the height of an homogeneous atmosphere, and shew that it is the same for whatever distance above the Earth's surface it is computed.

7. Describe the sea gage, and find the depth of the sea by means of it.

8. Find the density of the air in the receiver of an air pump after any number of motions of the pistons; shew that the defect of the mercury in the gage from the standard altitude decreases in geometrical progression. If a barometer be placed under the receiver, shew that the sum of the heights of the mercury in it, and in the gage, will be constant.

9. Define the centre of pressure, and find its position in a plane surface immersed in a fluid.

10. Investigate the equation whose roots determine the positions of equilibrium of a triangular prism, floating in a fluid with one angle immersed; if that angle be the vertical angle of an isosceles triangle, find it, when there is only one position of equilibrium.

11. Investigate the density of the air at a given height above the Earth's surface, and shew how the result may be applied to ascertain the heights of mountains.

12. In the common pump find the elevation of the water after one ascent of the piston, and after any number of ascents. If the piston do not descend as far as the fixed sucker, find the limit to the elevation of the water. Also find the pressure upon the piston during its ascent, both before, and after the water has got above it.

QUEEN'S COLLEGE, 1824.

1. SUPPOSE a given right cone to be just immersed in a fluid with its axis perpendicular to the surface, first with its base, then with its vertex downwards, required the pressure upon its whole surface in each case. The surface of a cone, not including the base, being equal to the circumference of its base, multiplied into half the length of its slant side, and the distance of its centre of gravity from the vertex two-thirds of its axis.

2. If a given isosceles triangle form one side of a vessel, half the perpendicular depth of which is filled with mercury, and the other half with water, required the line parallel to the horizon, which will divide the triangle into two areas, each of which shall sustain an equal pressure, the base of the triangle being coincident with the surface of the water.

3. Supposing a cylinder, given in magnitude and specific gravity, to be sustained in an erect position in three fluids of given specific gravity, required the depth to which it would sink in the lowest fluid: the specific gravity of the cylinder being greater than that of two of the fluids, and less than that of the other, and the length of the part of it immersed in the middle fluid given.

4. Compare the resistance to a given cylinder moving in a fluid in the direction of its axis, with the resistance to the same cylinder, moving with the same velocity, in a direction perpendicular to its axis.

5. A jet spouts from a tube in the side of a given cylindrical vessel filled with water, at an angle of 60°, and strikes the horizontal plane, on which the vessel stands perpendicularly, at the greatest possible distance from its base; required the height of the aperture.

6. If a given cylindrical vessel were to be supplied with water at a given rate, and when full to have an orifice of given area open in its base, required the lowest depression of the surface of the water, and the time of sinking to it, the supply of water being less than its efflux when the vessel is full.

7. If a cylindrical tube of given length, closed at one end, were let down into the sea in a vertical position, and when drawn up, the height to which the water has risen in it observed; required the depth to which it had descended. The weight of the atmosphere being supposed equal to that of 33 feet of sea-water. How must this tube be graduated, so that it may be used as a gage to measure depths in the sea.

8. Let a tube whose length is 37 inches, be filled with $21\frac{2}{3}$ inches of mercury, with 14 inches of water, and with air in its natural state in the remaining part; required the depth to which the mercury would subside after immersion, if the barometer were to stand at 30 inches

9. A barometer, having some air in the tube, stands at an altitude of (a) inches, being put under the receiver of an air pump, which contains (m) times as much as its barrel, after n turns it stands at an altitude of b inches, required the standard altitude and the quantity of air in the tube at first.

10. A receiver whose magnitude is (m) has two barrels connected with it; one of which, in magnitude (a), condenses, and the other, in magnitude (b), exhausts. They take their strokes alternately beginning with the barrel (a); required their effect after an infinite number of strokes.

11. The length of the gage of a condenser is 12 inches, and the space occupied by the air in it, after two descents of the sucker, is half its whole length; required the space which the air would occupy after the third descent of the sucker.

QUEEN'S COLLEGE, 1827.

1. A SQUARE is immersed vertically in a fluid with one corner coinciding with the surface: shew how the pressure upon it varies for different inclinations of the side to the surface.

2. A vessel of given height is filled with a fluid; where must an orifice be made in the side so that the issuing fluid may spout to a given distance on the horizontal plane?

3. A fluid is whirled round in a bucket with a given velocity; find the latus rectum of the paraboloid which the surface of the fluid will assume.

4. A given empty cylindrical vessel is immersed in a fluid to a given depth, and there fixed, an orifice being made in the bottom: required the time of filling to its greatest height.

5. A slender column of fluid oscillates in a cycloidal tube; shew that the oscillations are isochronous.

6. Shew how the centre of pressure may be found when the density varies according to any given law, and find it in a vertical square, one side coinciding with the surface, and the density varying as the depth.

7. The resistance on a cone moving in the direction of its axis : ditto on its base as 3 : 4; required the angle at the vertex of the cone.

QUEEN'S COLLEGE, 1828.

1. The weight of a vessel being given when empty, when filled with water of given specific gravity, and when filled with some other fluid; find the specific gravity of that fluid.

2. A rectangular flood-gate turns on a horizontal axis; find the position of the axis, that the water having risen to the top of the gate may just, by its pressure, be able to open it.

3. Compare the time of emptying a sphere with the time of emptying its circumscribing cube through a small orifice at the bottom.

4. A cylindrical tube of given dimensions, closed at one end and partly exhausted is immersed vertically to a given depth, when the height to which the water rises in it is observed. Hence find the quantity of air in the tube.

5. Construct the common pump, and find the height to which the water is raised by the two first ascents of the piston.

6. A tube of considerable length, but of small dimensions in other respects, is bent into the form of a circle and made to revolve about a vertical diameter. If 30° be occupied by water the rest being a vacuum; required the angular velocity that the lowest quadrant may be deserted by the water.

QUEEN'S COLLEGE, May 1829.

1. A cone and a paraboloid of the same altitude, floating with their vertices downwards, have each $\frac{1}{4}$th of the axis above water; compare their specific gravities.

2. Compare the resistance on the arc of a plane curve moving in a fluid, in the direction of its axis, with the resistance on the base; and apply the formula to the case of a semicircle.

3. A floodgate moving about a vertical axis, is divided by it into two parts, one of which is a rectangle, and the other an inverted

right-angled triangle of the same altitude. Given the base of the triangle, find the width of the rectangle, so that the gate may just open by the pressure of the water, when it has risen to the top.

4. A parabolic vessel, terminated by the plane, at right angles to the axis, passing through the focus, empties itself by an orifice at the vertex. Compare the spaces described by the descending surface, in the first and last halves of the time.

5. A globe descends from rest by its gravity in a medium the resistance of which varies as $(\text{vel})^2$. Determine its velocity at any point, and the time of descending through a given space.

6. A small orifice is made in a vessel containing condensed air: determine the time of discharge till the density within equals that without.

7. A tube AB, open at A and closed at B, containing a piston of given weight and dimensions, between which and the closed end, air is included, is whirled about A in a horizontal plane. Given the whole length of the tube, also the length occupied by the air before and after the motion; to find the angular velocity.

8. What must be the dimensions of a balloon, the whole weight of which, with its appendages, is 800 pounds; that it may just rise two miles high; supposing air to have 10 times the specific gravity of gas under the same pressure; that 5 cubic feet of air at the earth's surface weighs 6 ounces; and that the density at the earth's surface is four times as great, as at the height of 7 miles?

QUEEN'S COLLEGE, MAY 1830.

1. A TRIANGLE is formed by drawing two straight lines from two given points in the diagonal of a square, to one of the opposite angles: the square is then immersed vertically to a given depth, so that one of the sides adjacent to the said angle is parallel to the surface; compare the pressures on the three sides of the triangle.

2. A flood-gate turns on a vertical axis, the area on one side of the axis being a right-angled triangle with the base lowest, and on the other side a rectangle, of the same altitude. Required the width of the rectangle, so that the gate may *just* open by the pressure of the water, when it has risen to the top.

3. Find the depth of a parabolic vessel which discharges that portion of the fluid that is above the horizontal plane passing through the focus, in the same time with that portion which is below the focus.

4. A cylindrical vessel of given dimensions and full of water is pierced at a given point; find another point in which if it be pierced, the fluid issuing from each orifice may fall on the horizontal plane on which the vessel stands, at the same point.

5. If an inverted hemisphere full of air be forced down, so as just to be immersed in mercury; construct the equation from which (x), the height to which the mercury rises within the hemisphere, is to be determined.

6. Explain the principle of the common pump, and find the altitude to which the water rises in consequence of the two first strokes.

7. Find the time in which the air rushing into an exhausted receiver through a given orifice will have acquired a given density within. The $(\text{velocity})^2$ varying as the pressure, and h being the height due to the initial velocity.

8. If water ascend and descend in the arms of a cylindrical canal, which are every where of the same diameter, and inclined to the horizon at any angles; determine the time of an oscillation.

ASTRONOMY.

TRINITY COLLEGE, 1822.

1. Give proofs of the Earth's spherical form. What is its figure more accurately?

2. Define latitude and longitude on the Earth's surface, and explain some of the methods of determining the latitude.

3. Supposing a chronometer set at Greenwich, time not to have varied, and to indicate $7^h\ 20'$ P. M., when by observation the time of day at a given place is $11^h\ 35'$ A. M. Required the longitude of the place from Greenwich.

4. By what phenomena are the periods of a day and a year ascertained? and what is the correct length of the latter, and what consequent adjustment of the civil year necessary?

5. What angle is the ecliptic inclined at to the equator? and what the magnitude of the different zones? Supposing the poles of the ecliptic and equinoctial to coincide, what effect would be produced upon the days and nights?

6. Required the Sun's place in the ecliptic on the 31st of May; and thence determine the Sun's declination and right ascension for that day.

7. Required the latitude of a place at which the Sun rises upon the N.N.E. point on the longest day.

8. Explain solar and lunar eclipses very briefly, and give the ecliptic limits for each, and mention the greatest number of each that can possibly occur in a year.

TRINITY COLLEGE, 1823.

1. What are the direct proofs of the Earth's motion on its axis, and of its annual motion round the Sun?

2. Point out the necessity and sufficiency of the Gregorian correction of the Calendar.

3. The ecliptic polar distance of γ Draconis, which is situated nearly in the solstitial colure, is about 15°. Supposing the diameter of the Earth's orbit to subtend an angle of $2''$ at the star, prove that its distance is at least 200,000 times greater than the distance of the Sun.

4. Project the diurnal path of a star orthographically upon the plane of the meridian; and prove upon the projection, that when the star is in the horizon,

$$\text{rad} : \text{cos. azim.} :: \text{cos. lat.} : \text{sin. dec.}$$

5. In a given latitude, on a given day, to find the time of Sun-rise.

6. Having given the latitude of the place, and the declination of the Sun, investigate the equation to the locus of the extremity of the shadow which an upright style casts upon an horizontal plane. Prove that it is a conic section, and find its dimensions.

7. Find an expression for the angle which the Earth's disc subtends at a planet, and shew that some of the necessary observations do not require very great accuracy.

8. How did the ancients observe the angular distance of a planet from the Sun? Shew that this can be found from proper observations, without the intervention of a third object, and perform the computation.

9. Assuming the true system, demonstrate the retrograde, and stationary appearances of the planet Venus.

10. If the distance of Venus from the Sun were to the Sun's distance from the Earth $:: 1 : \sqrt{5}$; prove that her greatest brightness would be at her greatest elongation.

11. Give all the steps of the method by which Kepler found various distances of the planet Mars from the Sun, and the angles at the Sun contained by those distances, and was induced to conclude, that the orbit was not circular but oval.

12. Describe some of the more accurate methods of finding a Meridian Line.

13. Find the quantity of refraction by the circumpolar stars, (Boscovich's method); the refraction being supposed to vary as the tangent of the apparent zenith distance.

14. Transform the expression for the aberration in R. A. of a known star to the form $m \,.\, \cos.(L \sim K)$ where L is the Sun's longi-

tude, and K his longitude when the aberration is a maximum and positive; and shew that the R. A. (A), and longitude K, must be in opposite quadrants.

15. The N. P. D. of every star having the same R. A. is equally affected by precession.

16. Give a solution of Kepler's problem.

17. Explain La Caille's method of determining the place of the apogee of the Sun's orbit; and shew how its place affects the duration of the seasons.

18. Investigate the heliacal rising of a known star at any distant period. Give the solution of the requisite triangles, assuming that the Sun is about 12° below the horizon when a star of the first magnitude rises heliacally.

19. To determine when the absolute equation of time from both causes is a maximum.

20. To find the longitude of a place by observing the difference of the times of the transits of the Moon and a fixed star; the same observations having been made at a place, the longitude of which is known.

21. Explain the principle of the method of ascertaining the Sun's parallax, by observing the transit of Venus, and the reason of its superior accuracy.

22. Assuming the value ($a - 2c + 3c \sin. l^2$) of the radius of curvature at a place in an elliptic meridian of the Earth, where latitude is l, a being the semi-axis major, and c the ellipticity; find the value of a portion of the meridian between two given latitudes; and supposing two such portions to be known from actual measurement, deduce expressions for the diameters of the earth.

TRINITY COLLEGE, 1824.

1. On what day of the week will the year 2000 begin?

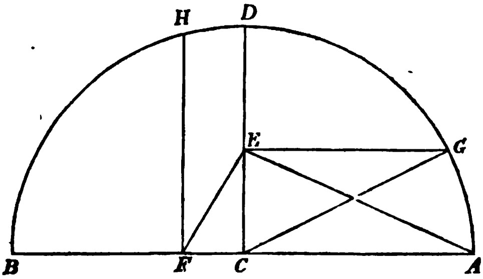
2. Explain the following astronomical terms:

Correction, Element, Anomaly, Equation.

3. Find the latitude of a place by observing two altitudes of the Sun at a given interval on the same given day.

4. Find, by observation, the time of the solstice.

5. CD is a radius perpendicular to the diameter AB of a circle;



EF, EG, FH, are perpendicular to AE, CD, AB respectively. Shew that if the angle BAE is made equal to the latitude of a place in the torrid zone, then, when the declination of the Sun in winter is equal to the latitude of the place, his azimuth at rising is the angle DCG, and the length of the day is $\frac{BH}{ADB} \times 24$ hours.

6. How must a horizontal plane be marked that the shadow of a vertical style may indicate the day and hour?

7. Deduce the rules for delineating the orthographic projection of a given portion of the globe on a plane parallel to the horizon of the place whose longitude and latitude are each $= 0$.

8. If D and d are the lengths of a degree of latitude and longitude in latitude 45°, shew that the equatorial is to the polar diameter of the Earth in the ratio $\sqrt{2} . d : D$.

9. In what latitude can twilight last all night but once in the year?

10. By means of the following logarithms, calculate the precise duration of that portion of a polar winter which is without twilight:

$\log. \sin. 18° = 9.4899824$ $\log. \sin. 23° 28' = 9.6001181$

$\log. \cos. 39° 6' = 9.8898877$ $\log. \cos. 39° 7' = 9.889785.$

11. Account for a planet appearing stationary; and find when Saturn will appear stationary, assuming his mean distance to be that of the Earth as 19 to 2.

12. Describe and account for the phænomena exhibited by the planet Venus in the course of a synodical revolution.

13. Assuming the necessary observations, find the place of the

node of the orbit of a planet, and its inclination to the plane of the ecliptic.

14. The periodic times of the Earth, of Venus, and of Mercury, being respectively $365^d\ 6^h\ 9'\ 11\frac{1}{2}''$, $224^d\ 16^h\ 49'\ 10\frac{1}{2}''$, and $87^d\ 23^h\ 15'\ 43\frac{1}{2}''$, shew that if a transit of Venus takes place, another may be expected at the same node after eight years, but that if it does not then occur, there cannot happen a transit at that node before 227 years more have elapsed; and find the intervals after which transits of Mercury at the same node may be expected to recur.

15. Measure the apparent diameter of the section of the Earth's umbra and penumbra at the distance of the Moon.

16. What is the age of the Moon when she rises about noon, and what is her age when she rises about midnight?

17. Prove, that at the full Moon next to the autumnal equinox, the Moon rises at nearly the same time for several successive nights; and shew what situation of the Moon in her orbit is most favourable to this phænomenon.

18. What are the three principal inequalities of the Moon's motion?

19. By observing the meridian altitudes and diameters of the Moon, at its greatest north and south latitudes, find its parallax.

20. Find the aberration of a given star in right ascension and declination.

21. From the mean anomaly calculate the true.

22. The Sun's parallax is $8''\ 73$, the Earth's diameter is 7912 miles, and the eclipses of Jupiter's satellites are seen by us $16'\ 26''$ later when he is in opposition, than when he is in conjunction. From these data deduce the velocity of light.

TRINITY COLLEGE, 1827.

1. DEFINE the following terms: zenith, nadir, rational horizon, azimuth, declination, right ascension, the latitude and longitude of a point on the Earth's surface, and also of a heavenly body; and illustrate the several definitions by a figure.

2. Explain the method of finding the right ascension of the Sun, and the obliquity of the ecliptic.

3. Given the latitude of the place and the Sun's declination, to find the time when he is on the prime vertical, and his altitude at that time. Apply the solution to the following case:

log. tan. dec.	$= 9.4548275$	log. sin. lat.	$= 9.8977604$
log. sin. dec.	$= 9.4378706$	log. cos.77°. 13′	$= 9.3444011$
log. cot. lat.	$= 9.8895736$	log. sin.20°. 17′	$= 9.5401012.$

4. To find the altitude and longitude of the nonagesimal.

5. The angle between any lines on the surface of a sphere is the same as that between their stereographic projections.

6. Construct a vertical south-east dial.

7. Given the lengths of two degrees of the meridian whose middle points are in two given latitudes; to find the compression.

8. Explain the causes on which the equation of time depends; and given the sidereal time $= 5^h\ 57'\ 56''$, find the mean time, supposing the mean Sun's right ascension when on the meridian $= 1^h\ 17'\ 6''$.

9. Define parallax, and explain the method of finding it by means of two observers on the same meridian, the Earth being supposed to be spheroidal.

10. Find the lengths of the tropical, sidereal, and anomalistic years.

11. Supposing the atmosphere to be homogeneous, and that m denotes the ratio of $\sin.I : \sin.R$ out of a vacuum into the air at the Earth's surface; then for a zenith distance z the refraction

$$= \frac{m-1}{\sin.1''}(\tan.z - \cdot 00125 \tan.z \sec.z^2) \text{ nearly.}$$

12. Find the precession in right ascension and declination of a given star; and shew in what case it is increased or diminished.

13. If the velocity of the Earth be in a finite ratio to that of light, shew that a ray proceeding from a heavenly body must strike the retina in a different point from what it would do if the eye of the spectator were at rest. Find the quantity and direction of the aberration.

14. Find the aberration in latitude and longitude of a given star.

15. In an elliptic orbit, where v denotes the true anomaly,

reckoned from the perihelion, u the eccentric, and $\dfrac{CS}{AC + BC} = c$, then will

$$v - u = 2\left(c \sin.u + \frac{c^2}{2}\sin.2u + \frac{c^3}{3}\sin.3u + \text{\&c. } \textit{ad infinitum}\right).$$

16. Given the geocentric, to find the heliocentric longitude and latitude of a planet.

17. Enumerate the elements of a planet's orbit; and explain the methods of determining them.

18. Explain the causes of the stationary and retrograde appearances of the inferior and superior planets, and point out the difference of the two cases as corroborative of the Copernican System.

19. To find the time, magnitude, and duration of a lunar eclipse.

20. Explain the method of finding the longitude by the observed distance of the Moon from the Sun or from a fixed star.

TRINITY COLLEGE, MAY 1828.

1. EXPLAIN the Ptolemaic hypothesis of the solar system. What phenomena would it account for and what not?

2. What knowledge of the motions of the heavenly bodies can we obtain without knowing the form and magnitude of the Earth? What additional information can we deduce from knowing those circumstances? How are this form and magnitude to be ascertained? Mention when and by whom any steps in this knowledge have been made.

3. Compare the times during which a circumpolar star moves eastward and westward.

4. Explain the changes produced in the appearance of the stars by the precession of the equinoxes. If a known star rose at sunset in former ages, how could we determine the period at which this happened?

5. Explain this method of adjusting a transit instrument so that,

(1). The line of vision shall be perpendicular to the axis.
(2). The axis shall be horizontal.
(3). The motion shall be in a meridian plane.

and supposing the radii of the two ends of the axis to be unequal, and the level to be incorrect, shew how the proper corrections may be made.

6. Explain by what observations Bradley detected the aberration of light, and how he distinguished its effects from those of a nutation of the Earth's axis.

7. If the fixed stars have an annual parallax, its effect on the place of a star may be fouud from the formulæ for aberration, by supposing the longitude of the Earth to be 90° greater than it really is.

8. If a be the right ascension, and δ the north polar distance of a star, Δ its co-latitude, ω the obliquity,

$$\sin.^2\frac{\Delta}{2} = \sin.\{\tfrac{1}{2}(\omega + \delta) + M\}\sin.\{\tfrac{1}{2}(\omega + \delta) - M\},$$

where $\sin.^2 M = \sin.\omega \sin.\delta \sin.^2 \tfrac{1}{2}(90 - a)$.

Apply these formulæ to α Arietis,

$$a = 1^h\ 56^m\ 45^s{\cdot}9.$$
$$\delta = 67^\circ\ 25'\ 1''{\cdot}7.$$
$$\omega = 23^\circ\ 27'\ 46''{\cdot}3.$$

log. sin.17° 52′ 12″ ·3 = 9·4869392.
log. sin.23° 27′ 46″ ·3 = 9·6000517.
log. sin.27° 34′ 11″ ·7 = 9·6654221.
log. sin.30° 24′ 15″·75 = 9·7042361.
log. sin.40° 1′ 11″ ·3 = 9·8082463.
log. sin.63° 18′ 36″ ·3 = 9·9510705.
log. sin.67° 25′ 1″ ·9 = 9·9653546.

9. Shew how we may find the time at which the aberration of a given star in declination is 0; and apply this to α Arietis, of which

the latitude = 9° 57′ 37″, log. sin. = 9·23796.
the angle of position = 20° 39′ 52″, log. tan. = 19·57653.
65° 21′ 50″, log. tan. = 10·33857.

10. Prove the formula for the effect of precession in R. A.

$$da = 50''(\cos.\omega + \cot.\delta \sin.\omega \sin.a).$$

How are the stars situated for which this correction is 0?

11. A known star is observed to pass the prime vertical to the east and west, and the intervening time is observed; find the latitude.

12. In the last question supposing an error of 1 second of time in the observed interval; find the corresponding error in the latitude deduced.

13. Find the position of the ecliptic at any instant, the angle which it makes with the horizon, and the point where it cuts it.

14. Explain under what circumstances Venus will be direct and retrograde; and her distance being ·723 the Earth's, find how long after conjunction she will be stationary.

15. In an orbit of which the eccentricity is e, the true anomaly v, the eccentric anomaly u; if $\sin.\,\varepsilon = e$, the equation of the centre will be greatest when

$$\sin.\tfrac{1}{2}\,(v - u) = \sin.u\,\frac{\sin.\frac{1}{2}\varepsilon}{\cos.^{\frac{1}{4}}\varepsilon}.$$

16. If u be the distance of the Sun from the solstice in R. A., y the defect from the solstitial declination, ω the obliquity,

$$y = \tan.^2\tfrac{1}{2}u\,\sin.2\omega - \tfrac{1}{2}\tan.^4\tfrac{1}{2}u\,\sin.4\omega + \&c.$$

17. Supposing S' to be the true Sun moving unequally in the ecliptic, S'' a point moving equably in the ecliptic, S''' a point moving equably in the equator in the same time, explain the orders of arrangement which S', S'', S''' will assume in the course of a year.

18. Shew that on any hypothesis of the density of the atmosphere, the refraction

$$= \lambda D\,\frac{1 - \beta t}{1 + at}\,(\tan.Z - m\,\tan.Z\,\sec.^2 Z),$$

and explain how m is found.

19. A ship sails perpetually due N. E.: what will be the length of its course before it reaches the pole, and the area included?

20. What day of the week was April 27, 1769? What are the rules for this calculation?

21. The sphere being orthographically projected on the horizon of a given place, give rules for describing the meridians and parallels.

22. A ladder leaning against a south wall happens to be inclined to the horizon at an angle equal to the latitude of the place: shew that the shadow will occupy the same situation every day at the same

hour; and determine the position of this shadow on the wall and on the ground.

23. Shew how the difference of longitude of two places may be determined by observing the transit of a star and of the Moon at the two places. How far does this method depend on the accuracy of the lunar tables?

24. Find the place of the node of a planet's orbit from observation.

25. Find the inclination of the Sun's equator to the ecliptic, and the time of the Sun's rotation.

TRINITY COLLEGE, JUNE 1829.

1. IN what respects does the received system of the universe differ from those supposed by Ptolemy and by Tycho Brahe?

2. Give a brief account of the solar system, stating what is known concerning the relative distances of the planets from the Sun, and the satellites from their primaries.

3. Explain the distinction between solar and sidereal time. Which of them is kept by the Observatory clock, and for what reason?

4. What is the nature of the observations made at the Observatory, and what is the object of them?

5. At what time of the day, when the Sun's declination is 20° 16′, will the length of the shadow of a vertical gnomon be just equal to the height of the gnomon itself?

6. Explain the method of finding the latitude of a place from two equal altitudes of the Sun observed on the same day, including the correction for a change of declination during the interval.

7. What is meant by the first point of Aries? How does it happen to be in the constellation Pisces? How is its place at any time determined?

8. The right ascension of a star being $13^h\ 14'\ 18''$, its declination $10^\circ\ 4'\ 31''$, and the obliquity of the ecliptic $23^\circ\ 28'$; required the star's latitude and longitude.

9. If z represent the zenith distance of a star, t the excess of the height of the thermometer above 50°, b the height of the barometer, the refraction is

$$\frac{b}{29\cdot6}\cdot\frac{1-\beta t}{1+at}\,57''\cdot82\,(\tan.\,z - n\tan z\,\sec.z^2).$$

How was this formula obtained?

10. Explain the effect of parallax. Which of the heavenly bodies does it affect? How is the quantity of it determined?

11. Investigate the alteration caused in the right ascension and declination of a star by the precession of the equinoxes.

12. Explain the phenomenon of the aberration of light.

13. Explain the method used to find the time, magnitude, and duration of a lunar eclipse.

14. Explain the method of constructing a sun-dial to be affixed to a given vertical wall.

15. What kind of sun-dial should be used at the equator?

16. Explain the method of drawing a meridian line.

17. In latitude 39° 54′ N. longitude 35° 30 W. the altitude of the Sun's lower limb, on the 7th of May, at $5^h\ 30'\ 32''$ P. M. per watch, was observed to be 15° 40′ 57″; required the error of the watch.

Refraction............ 3′ 19″.

Parallax .... 0′ 9″.

⊙'s semi-diam......15′ 10″.

⊙'s dec. 0^h, May 7...17° 5′ 24″ N.

ditto.............. 8...17° 21 34″.

18. On the 15th of May, in altitude 33° 10′ N., and longitude 18° W., about 5 o'clock, A. M. the Sun was observed to rise E. by N. Required the variation of the compass.

TRINITY COLLEGE, MAY 1830.

1. GIVE a brief sketch of the motions which are observed to take place among the heavenly bodies.

2. Define the terms *latitude* and *longitude*, as applied to places on the surface of the Earth, and enumerate the methods employed to determine them.

3. What is meant by an error in the line of collimation of a transit telescope? How is it detected?

4. Detail the method used to draw a meridian line, with the correction for the change in the declination of the Sun during the interval of the observations.

5. The altitude of the Sun's centre was observed one afternoon in London, (latitude 51° 32) to be 38° 19′, its declination being at that time 19° 39′ N. Required the hour of the day.

6. The latitude of a Star being 16° 3′ S. Its longitude 2^s 25° 52′, and the obliquity of the ecliptic 32° 28′, required the sidereal time of its passage over the meridian.

7. It is required to calculate the time of the Sun's rising, in London, from the following data:

Sun's declination = 17° 33′ Sun's semidiameter = 16′

Horizontal refraction = 32′ Parallax = 9′.

8. Prove that for small zenith distances the refraction varies nearly as tan.z.

9. Explain the nature of parallax, and determine the amount by which it affects the apparent diameter of the Moon.

10. Explain what is known, as a matter of fact and observation with respect to the precession of the equinoxes and nutation of the Earth's axis.

11. Find the effect of aberration on the latitude of a Star. Show what it becomes when the Star is supposed to be at the pole of the ecliptic.

12. Explain the terms *mean* and *true anomaly*, and show how the latter is to be deduced from the former, in an elliptic orbit of small eccentricity.

13. Explain the term *equation of time,* and show on what it depends.

14. Given three distances of a planet from the Sun, and the corresponding arguments of latitude, to find the place of the perihelion, and the true anomaly at the first observation.

15. Detail the *Lunar Inequalities,* in the order in which they were discovered.

16. Show how to find the time, magnitude, and duration, of a lunar eclipse.

17. Determine the number of solar and lunar eclipses, which may, or which must, take place in one year.

18. From the following observed values of the Sun's declination, at noon, find, by interpolation, the time of the solstice.

June 19	23°	26′	39″
20	23	27	23
21	23	27	42
22	23	27	37.

St. JOHN'S COLLEGE, June 1820.

1. If two sides of a spherical triangle be equal to a semicircle, the arc drawn from the vertex bisecting the base is a quadrant.

2. If two sides of a spherical triangle be less than a semicircle, the angle opposite the less side is less than 90 degrees.

3. If a great circle be stereographically projected, the radius of the projection is the secant, and the distance of its centre from the centre of the primitive is the tangent of the inclination of the circle to the primitive.

4. Having given the Sun's declination and diameter, and the latitude of the place; find how long the disk is in rising.

5. Find the absolute quantity of refraction for considerable altitudes according to Boscovich's method.

6. On a certain island the Sun was observed to be vertical when on the meridian. The declination was found from the tables to be *D* degrees; and the time of sunrise at Greenwich to be *T*, and of sunset *t* hours before that observed by the chronometer. Find the distance and bearing of the place from Greenwich.

7. Having given the Sun's rising, amplitude, and altitude when on the prime vertical, determine the latitude of the place.

8. The Sun's declination which is given, is greater than the co-latitude which is also given. Determine the nature and dimensions of the curve which is traced out by the end of the shadow of a vertical stick on an horizontal plane.

9. Determine, from three observations of a spot on the Sun's disk, the inclination of the Sun's equator to the ecliptic, the longitude of the ascending node, and the time of the Sun's revolution on its axis.

10. When a planet is stationary, the horary change in elongation is to the horary change in annual parallax as the periodic time of the planet to the periodic time of the Earth.

11. Find the shape and dimensions of the Moon's penumbra.

12. Explain how the Sun's parallax may be found by the transit of the inferior planets.

13. By the effect of aberration, a Star's latitude is least when the Sun's place is 90 degrees before the Star's place in the ecliptic, and from that time the increment in latitude varies as the versed sine of the Sun's longitude, reckoned from that point.

14. The longitude is least when the Sun is in syzygy with the Star, and from that time the increment in longitude will vary as the versed sine of the Sun's longitude reckoned from that point.

15. If a Star be situated very near the pole of the ecliptic, the decrement of the apparent longitude varies as the versed sine of the Sun's longitude, reckoned from a point 90 degrees behind the Star's place in the ecliptic.

ST. JOHN'S COLLEGE, MAY 1823.

1. GIVEN the two sides and included angle of a spherical triangle; find the third side, and adopt it to logarithmic computation.

2. If the sides AC, BC of a spherical triangle be produced to D and E, so that $\tan.\frac{AC}{2} : \tan.\frac{DC}{2} :: \tan.\frac{EC}{2} : \tan.\frac{BC}{2}$, the triangles have equal areas.

3. Mention some phenomena that can be accounted for by a real annual and diurnal motion of the earth, but not by a motion of the sun and heavens.

4. Explain the vicissitudes of the Seasons, and the cause of their different lengths, writing them down in the order of duration.

5. If two known stars, observed at the same time in the same vertical, come to the meridian at intervals h and h' respectively, find the latitude of the place.

6. Draw a meridian from observing the shadow of a vertical gnomon, when the Sun is near the solstices, and correct for a small change in declination during the observation.

7. At a given place, stars situated in the equator appear to rise quickest.

8. Construct a horizontal dial, and shew how such a dial, calculated for latitude (a), must be graduated so as to serve in latitude (b).

9. Given the altitude through which the Sun's centre is raised by refraction, find the corresponding diminution of its horizontal diameter.

10. If z and z' be two zenith distances of a planet observed at places on the same meridian whose latitudes are l and l',

$$\text{the horizontal parallax} = \frac{z + z' - (l + l')}{\sin.z + \sin.z'}.$$

11. The annual precession in declination of a star is $50'' \cos.A \times \sin.I$, A and I being the right ascension and obliquity.

12. Whence does the latitude of the Sun arise? find its greatest value, supposing the Earth, Sun, and Moon, the only bodies in the system.

St. JOHN'S COLLEGE, 1826.

1. The sines of the arcs, drawn from the angles of a spherical triangle perpendicular to the sides, are to each other, inversely as the sines of the sides on which they fall.

2. Bisect a given spherical triangle by an arc drawn from one of the angles.

3. Prove Napier's rules, when the complement of the hypothenuse is made the middle part.

4. In the stereographic projection, find the radius of the projection of a circle whose plane is perpendicular to the primitive.

5. By what observations does it appear that the diurnal motion of the heavens is circular?

6. Explain how the formulæ of refraction are corrected for temperature and atmospheric density.

7. Given the difference in the lengths of the day and night when the Sun is in the equator; find the horizontal refraction; neglecting the Sun's motion.

8. If P be horizontal parallax of the Moon, p any other parallax when the apparent zenith distance is $z + p$, shew that

$$P = \frac{\sin.z}{\sin.1'} \cdot \sin.P + \frac{\sin.2z}{2\sin.1''} \cdot \sin.^2P + \&c.$$

and explain distinctly that part of the proof in which an arc expressed in parts of the radius is converted into an equivalent angle.

9. Find the major and minor axis of the ellipse which a given star appears to describe in consequence of aberration.

10. Find, by observations made at any place, the bearing of a distant terrestrial object.

11. From the given difference of the times at which two known stars reach the same altitude, find the latitude of the place of observation.

12. Having given two observed declinations of the Sun and the Sun's right ascension; explain why the method is practically less accurate than that of Flamstead.

13. Given the Sun's greatest and least motion in longitude; find the eccentricity of the Earth's orbit.

14. State distinctly the steps by which a person may find his latitude, longitude, and the time; supposing him possessed of the proper instruments and tables, and a good chronometer which he has neglected to wind up.

St. JOHN'S COLLEGE, May 1827.

1. In the solution of oblique-angled spherical triangles, explain in what cases there is an ambiguity.

2. Given two sides of a spherical triangle, and the arc bisecting the included angle, and terminating in the opposite side; solve the triangle.

3. If Δ be the length of an arc in Mercator's projection, between the equator and latitude (λ), shew that

$$\tan.\lambda = \tfrac{1}{2}(\varepsilon^{\Delta} - \varepsilon^{-\Delta}).$$

4. In a given latitude it is observed, that when the Sun is due east, the difference between his altitude, and the hour-angle from six, is a maximum; find the time of observation, and day of the year.

5. A horizontal dial being constructed for a latitude (λ), is fixed in another (λ'); find when the error will be the greatest.

6. Shew that the equation of time vanishes four times a year.

7. At a given place, find when the Sun's motion in azimuth : the motion in altitude :: radius : sin. zenith-distance, and determine the highest latitude where this can occur on a given day.

8. Find the annual parallax in longitude of a heavenly body, and shew that it is greatest when the aberration is least, and conversely.

9. A comet is observed to be in its perihelion 27 days after passing its node, and 5 days after that to be in its node again. Given the perihelion distance, shew how the position of the axis of the orbit, and the comet's distance at the times of observation may be found.

10. Two towns *A* and *B* are so situated, that when it is noon on a given day at *A*, the Sun is just rising at *B*. Given the latitudes of *A* and *B*, find the latitude of a place equally distant from both, and Δ° west of *A*.

11. A person sees a new comet, and, having no instrument with him, finds, by stretching a thread before it in one direction, that it

appears in the same line with two known stars, *a* and *b*; and by changing the direction of the thread with two other known stars, *c* and *d*. Hence, find the comet's place, the longitudes of *a* and *b* being supposed equal, and also of *c* and *d*.

12. Explain the method of finding the Sun's parallax by means of a transit of Venus, and give the reasons of its superior accuracy.

ST. JOHN'S COLLEGE, MAY 1828.

1. GIVEN the angles *A*, *B* and the side *a* of a spherical triangle, find the remaining sides and angles.

2. The angles of a spherical triangle form a progression whose common difference is 45° and the area : the sphere :: 7 : 48; find the sides of the triangle.

3. Find the radius and place of the centre of the ecliptic projected on a parallel of latitude, the eye being situated in the pole of the equator.

4. If a known star appear due east two sidereal hours after rising, at what time after will it appear south?

5. The meridian plane is taken for the vertical passing through two given stars, which are supposed to have the same right ascension, find the error in the plane of the meridian, supposing the star's right ascension to differ by a very small angle α.

6. If two stars with known declinations rise together, but cross the meridian after a given interval, find the latitude of the place.

7. In what latitude does the shadow at 8 o'clock in the morning move at the same rate on the horizontal and on the vertical south dial?

8. Find the aberration in declination of a given star, and shew how the coefficient of aberration may be determined from observations made on the declination of a star in the solstitial colure.

9. Determine the parallax of a heavenly body by observations made out of the plane of the meridian.

10. Three observations are sufficient in theory for determining the elements of a planet's orbit.

11. If a superior planet which describes a circle in the ecliptic appear stationary, find the difference of heliocentric longitude of the

earth and planet, and shew that this could not take place unless the earth moved.

12. If the heliocentric latitudes of a planet taken at equal intervals are 5° 16′ 30″, 5° 17′ 14″, 5° 17′ 33″, 5° 17′ 28″; find the inclination of the planet's orbit to the ecliptic.

ST. JOHN'S COLLEGE, JUNE 1829.

1. SHEW that in the solution of oblique-angled spherical triangles there can only arise four cases, and write down the equation for each; also *prove* the following one,

$$\cot. A \sin. C = \cot. a \sin. b - \cos. C \cos. b,$$

and by means of a subsidiary angle deduce from it any one of the quantities, in a form adapted to logarithms, when the other three are given.

2. Prove Napier's rules for the solution of right-angled spherical triangles, when the complement of an angle is middle part; and explain under what modifications they are applicable to the solution of quadrantal triangles.

3. Determine the time by an observed altitude of the Sun, and correct it for a small error in the altitude; and hence find the longitude of a ship that is provided with chronometers. In what respects does a chronometer differ from a common watch?

4. Define the angle of position of a star, and compute it from the right ascension and declination.

5. Find the Sun's azimuth at setting, and shew how the result may be applied to determine the variation of the compass. Is the deviation of the magnetic needle from the north always nearly the same at the same place?

6. By what observations is the existence of refraction proved? What is its effect on the heavenly bodies? Correct the mean refraction for a change in the pressure and temperature of the atmosphere.

7. To find the aberration in right ascension and declination of a given star, and the time of the year when it vanishes in each case.

8. If n and L represent the height of the nonagesimal and its longitude, l and λ the longitude and latitude of a star, P the horizontal parallax; prove that the parallax in longitude

$$= P\frac{\sin.n \sin.(L-l)}{\cos.\lambda},$$

parallax in latitude $= P\{\cos.n \cos.\lambda - \sin.n \sin.\lambda \cos.(L-l)\}$.

9. To find the longitude of the perihelion, and the time of the Earth's passing through it.

10. Shew that the planets are opaque and spherical, and that they do not revolve round the Earth; also find the longitude of the node of a planet's orbit, from observations made on the planet when in its node.

11. To find Jupiter's distance, from an observed eclipse of a satellite. Mention some of the peculiarities in the motions of Jupiter's satellites, and the Astronomical discoveries to which they have led.

12. Explain the Moon's phases; account for the faint light observed on the Moon's disk when in conjunction, and also when suffering an eclipse.

ST. JOHN'S COLLEGE, JUNE 1829.

1. IN a spherical triangle whose sides are a, b, and included angle C, prove that half the spherical excess

$$= m \sin.C - \frac{m^2}{2}\sin.2C + \frac{m^3}{3}\sin.3C - \&c., \text{ where } \tan.\frac{a}{2}\tan.\frac{b}{2} = m.$$

2. A plane triangle whose sides are a, b, c is placed in a sphere radius $= r$; prove that the angle between the arcs of great circles of which a and b are the chords, will be 90°, if

$$r = \frac{ab}{\sqrt{\{2(a^2+b^2-c^2)\}}}.$$

3. If three stars have the same altitude, the difference of azimuth of any two is twice their angular distance as seen from the third.

4. If z, z', be the zenith distances of a star when in the same vertical on different sides of the meridian; prove that $\tan.\frac{z}{2}.\tan.\frac{z'}{2}$ is the same for all stars that have the same declination.

5. If BC represent the substyle, AC the six o'clock hour-line of a horizontal dial; and if AB be any line placed between them so that $BC.\sin.\text{lat.} = AC$, M its middle point, H a point in MB such that $MH = AM.\tan.(45 - h)$; prove that if CH be joined, it is the hour-line corresponding to the hour-angle (h); and explain the superiority of this as a practical method of constructing horizontal dials.

6. Suppose the latitude and longitude of a place on the Earth's surface are given; investigate formulæ for finding the latitude and longitude of another place very near the former, whose bearing and distance are known.

7. The stereographic projection of any diametral section of an oblate spheroid, on the plane of the equator, is a circle; the eye being placed in the pole; find its centre and radius.

8. When the equation of time is a maximum, the Sun's longitude (l) must be derived from the equation

$$\cos.\omega\{1 + e\cos.(l - \alpha)\} = (1 - e^2)^{\frac{3}{2}}(1 - \sin.^2 l \sin.^2\omega),$$

where (α) denotes the longitude of the perihelion.

9. Investigate the formula of lunar nutation in right ascension, and find the longitude of the Moon's nodes when it is a maximum.

10. The geocentric latitudes of an inferior planet at two successive conjunctions with the Sun, separated by an interval $\frac{S}{2}$, were observed to be β and β', the planet in both cases being nearly 90° from its node; if i be the inclination of its orbit, shew that

$$\cos.i = \left(\frac{S + P}{S}\right)^{\frac{2}{3}} \cdot \frac{\sin.(\beta - \beta')}{\sin.(\beta + \beta')},$$

where P equal one year.

11. To explain the physical cause of solar precession, and to investigate its quantity for any given time.

12. Shew that the attraction of the perimeter of an equilateral

spherical triangle on the centre of the sphere

$$= \sqrt{\left\{6 \tan.^2\frac{a}{2}\left(3 - 4 \sin.^2\frac{a}{2}\right)\right\}},$$

where (a) = length of its side; the attraction of a quadrant of a great circle being reckoned unity, and force $\propto \frac{1}{D^2}$. Explain the result when $a = 120°$.

ST. JOHN'S COLLEGE, MAY 1830.

1. INVESTIGATE the properties of the supplemental triangle.

2. Prove Napier's analogies.

3. When the vertical plane, in which a transit instrument moves, nearly coincides with the meridian, find the deviation by means of the observed superior and inferior transits of the same circumpolar star. What other errors affect the adjustment of a transit? How are they found out and corrected?

4. Find the right ascension of the Sun and the obliquity of the ecliptic.

5. Find the length of a tropical, a sidereal, and an anomalistic year.

6. Find the refraction considering the atmosphere homogeneous. Determine the coefficient of refraction.

7. Determine the parallax of a heavenly body by observations made in the plane of the meridian. How are the effects of parallax to be extricated from those of refraction?

8. The aberration of a star $= 20''\cdot 25 \,.\, \sin.$ Earth's way. Find the aberration of a star in right ascension.

9. Investigate the relation between the true and mean anomaly. Find the equation to the centre in orbits of small eccentricity.

10. Find the node of a planet's orbit from observations made on the planet in its node.

11. Find the longitude of a place by the observed distance of the Moon from the Sun, or from a fixed star. Are any other observations necessary?

Explain the method of finding small differences of longitude by means of signals made on the Earth's surface.

12. Construct a dial on a plane inclined at any given angles to the meridian and horizon.

Of what nature is the error in the time shewn by a dial? What are the sources of the error? When does it vanish?

13. How does the projection of a small portion of the Earth's surface near the edge of the projection differ from that of the same portion near the centre, in the two projections commonly used?

ST. JOHN'S COLLEGE, MAY 1830.

1. IN a spherical triangle where a, b, C are given, prove that c may be found from the formulæ

$$\sin.^2\frac{\phi}{2} = \sin.a \sin.b \cos.^2\frac{C}{2},$$

$$\sin.^2\frac{c}{2} = \sin.\frac{a+b+\phi}{2}\sin.\frac{a+b-\phi}{2};$$

explain why these formulæ are inconvenient when the side c is nearly 180°, and investigate similar ones suited to that case.

2. If α, β be arcs drawn from the right angle C of a spherical triangle respectively perpendicular to, and bisecting the hypothenuse; prove that

$$\cot.\alpha = \sqrt{(\cot.^2a + \cot.^2b)},$$

$$\tan.\beta = \frac{\sqrt{(\sin.^2a + \sin.^2b)}}{\cos.a + \cos.b}.$$

3. The hour-angle of a star, whose declination is δ, when seen in a certain vertical plane is h, and when next seen in the same vertical plane its hour angle is h': prove that tan. latitude of the place

$$= \frac{\cos.\frac{h+h'}{2}}{\cos.\frac{h-h'}{2}}\tan.\delta.$$

4. The angle between the shadow on a horizontal dial, and the position it would occupy if the style were vertical, is at any time the complement of the angle between the vertical circle passing through

the Sun, and that which cuts his declination circle at right angles; shew that at the equinoxes this angle is the greatest at 3 o'clock.

5. Given the latitudes of two places, find the radius and position of the centre of the stereographic projection of the parallel passing through one, upon the horison of the other.

6. The angular motion of the shadow of a vertical gnomon on a horizontal plane will be the greatest, when its length

$$= a\sqrt{\left(\cot.\frac{\alpha^2}{2} - 1\right)};$$

a being the length of the gnomon, and α the altitude of the Sun when on the prime vertical, on the given day.

7. How is it shewn that the diameter of the Earth's orbit subtends an evanescent angle at the fixed stars? If the fixed stars have an annual parallax, shew that the formulæ for aberration three months after the present time will represent its effects on their places.

8. The great circles in which a circumpolar star's motion seems to lie at three points of its course, are produced to meet, and the points of intersection are found to have the same latitude λ; if δ be the declination of the star, prove that sin. obliquity

$$= \sqrt{(\sin.^2(\delta - \lambda) - \cos.^2\lambda \cos.^2\delta)}.$$

9. In a given latitude, and on a given day, find how long the Sun will be in rising, and where and when he will rise. Also determine that star which rises in a given point of the horizon at the same instant as the Sun's centre.

10. Describe the repeating circle, and its use in finding the angular distance between two objects by the method of crossed observations.

11. To calculate the places on the Earth's surface where a given solar eclipse is visible.

12. If a ship sail in the shortest course from a place in latitude l, to a place in the equator, prove that in latitude l', where $\cot.^2 l' = \cot.^2 l + \operatorname{cosec.}^2 l$, she will be nearer the end than the beginning of her course by a distance $= \frac{an}{b} - b$; a and b being the semi-axes of the terrestrial spheroid, and n the length of the normal at starting, at which time also the ship's course is perpendicular to the meridian.

CORPUS CHRISTI COLLEGE, May 1830.

1. Enumerate the arguments by which the diurnal rotation of the Earth round its axis, and its annual motion round the Sun, are established.

2. Find the length of the normal at any point of the terrestrial meridian in terms of the latitude; and thence, by means of a seconds' pendulum determine the compression. The Earth being supposed to be an oblate spheroid of small eccentricity.

3. Define the Geodesic line, and shew that it is the shortest line that can be drawn on the Earth's surface between any two points of its course.

4. Explain the nature and use of the common vernier.

5. Given two altitudes of the Sun, and the time between; find the latitude and hour-angle.

6. Prove Brinkley's formula for the mean refraction, and shew that it is reducible to the same form as Bradley's.

7. The refraction being $= m \,.\, \tan.z + m' .(\tan.z)^3$; give Lacaille's method of computing the coefficients.

8. Find the effects of parallax on the hour-angle, and declination of a heavenly body.

9. Define the terms, Precession and Nutation, and find the effects of the former on the right ascension and declination of a given star.

10. Find the longitude of the perihelion, and the time of the Earth's passing through it.

11. If $\theta =$ the true anomaly,

$n.t =$ ... mean ....

$\varepsilon =$ ... eccentricity. Then, ε being small;

$$\theta = n.t + 2.\varepsilon \sin.nt + \frac{5.\varepsilon^2}{4}.\sin.2.nt \text{ nearly.}$$

12. Supposing a Comet to move in an orbit of great eccentricity;

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investigate the following formula for determining the distance (ϱ) of the Comet from the Sun, corresponding to given anomalies;

$$\tfrac{1}{4}.\text{h. l.}\left\{\frac{\varrho}{a.(1-\varepsilon)}\right\}$$

$$= \sin.\theta.\sin.\varphi - \tfrac{1}{2}.\sin.2\theta.\sin.2\varphi + \tfrac{1}{3}.\sin.3\theta.\sin.3\varphi - \&\text{c.}$$

where $\theta = \frac{1}{2}$. sum of the true and eccentric anomalies,
and $\varphi = \frac{1}{2}$ their difference.

13. Define the equation of the centre, and shew that in the Earth's orbit, its greatest value is nearly equal to twice the eccentricity.

14. Given three distances of a planet from the Sun, and the corresponding arguments of latitude: find the place of the perihelion, and the true anomaly at the first observation.

15. Explain the phases of Venus. State also M. Schroeter's reasons for supposing that her atmosphere is much denser than that of the Moon.

16. Find the time, magnitude, and duration of a lunar eclipse.

17. The orthographic projection of the rhumb line on the plane of the equator is an hyperbolic spiral.

18. Construct an horizontal dial for a given latitude, and determine the position of the last hour-line.

19. Given the Sun's altitude; find at what angle a straight rod must be inclined to the horizon, that the length of its shadow may be the greatest possible.

20. Describe fully the lunar method of determining the longitude at sea. Give also the analytical process adopted by Borda, for determining the true distance.

21. Given, $\tan.^{-1}\ 10 = 1{\cdot}4711276$,
$\tan.^{-1}\ 11 = 1{\cdot}4801364$,
$\tan.^{-1}\ 13 = 1{\cdot}4940244$,
find $\tan.^{-1}\ 11{\cdot}63$ by the method of interpolations:

22. Find the time of year when the duration of twilight is shortest.

23. Whence does it appear manifest that the Moon is surrounded by a much rarer atmosphere than that of our Earth? From what circumstance did some of our astronomers formerly deny the existence of an atmosphere at the Moon?

24. Supposing the Earth's orbit to be a logarithmic spiral, find the equation to the aberratic curve.

25. Explain the method of least squares as given by Gauss; and show that it gives the most probable values of the unknown quantities.

26. Determine the Sun's parallax by means of a transit of Venus over the Sun's disk.

CAIUS COLLEGE, MAY 1830.

1. $F \propto \frac{1}{D^{\frac{4}{3}}}$. Find the time of a body descending in a straight line from a given point to the centre of force.

2. $F \propto \frac{1}{D^{\frac{7}{2}}}$. A body is projected from any point in any direction with a velocity equal to that from infinity. Find the equation to the curve described, the position of the apse, and the whole angle described before the body falls into the centre.

3. A body is revolving in an ellipse, round the focus, and at the extremity of the axis-minor half the force is suddenly taken away; determine the alteration which takes place in the form of the orbit.

4. Prove that the force by which the cissoid may be described round a centre of force in the cusp varies as $\frac{(\text{cosec}.\theta)^2}{r^3}$, where θ is the angle which the radius vector (r) makes with the axis.

5. Find the time of an oscillation in a circular arc, when the force in the direction of the curve varies as $\frac{\theta}{\sin.\theta}$, θ being the distance from the lowest point.

6. A body is describing a circular arc, and acted upon by a force tending to a given fixed point without the circle; determine its velocity at any point.

7. The particles of a cone attracting with a force which varies as $\frac{1}{D^2}$, compare the attraction on a particle in the vertex with that on a particle in the centre of the base.

8. Find the centre of pressure of a semi-parabola immersed vertically in a fluid, with its vertex just coincident with the surface and its base parallel to it.

9. A vessel in the form of the hyperboloid of revolution has its axis vertical and vertex downwards; find the time of emptying it through a given small orifice in the vertex.

10. A given paraboloid contains a given quantity of fluid; determine the angle at which its axis may be inclined to the vertical before the fluid runs out.

11. A fluid mass has a given uniform motion of rotation, and its particles are attracted to two constant centres of force in the axis of rotation. Determine the form which its surface will assume.

12. A body oscillates in a circle, in the medium of which the resistance varies as the velocity; find the time of one of its small oscillations.

13. Find the lengths of the day and night, when

$$\frac{\cos.(l-\delta)}{\cos.(l+\delta)}=\frac{\sqrt{2}+1}{\sqrt{2}-1},$$

l being the latitude of the place and δ the Sun's declination.

14. The Sun's azimuths at the beginning and end of twilight are supplements to each other on the day of shortest twilight.

15. Prove that the times in seconds in which the Sun will pass the horizontal aud vertical wires of a telescope are respectively

$$\frac{D''}{15 \cos.\ \text{lat.} \sin.\ \text{dec.}} \text{ and } \frac{D''}{15 \cos.\ \text{dec.}},$$

D being the apparent diameter.

16. A style projects vertically from the top of an upright cone; find the path described by the extremity of its shadow on the surface of the cone.

OPTICS.

TRINITY COLLEGE, 1820.

1. Given the distance of the focus of incident rays from the centre of a given spherical reflector; find the distance of the geometrical focus of reflected rays from the centre, when they are incident nearly perpendicularly.

2. Given the position of an object placed between two plane reflectors inclined at a given angle; find the total number of images, and apply it to the case where the angle of inclination is equal to 11° 15′.

3. A straight line is placed before a concave spherical reflector, at the distance of one-third of its radius from the surface; find the dimensions of the curve formed by its image, the radius of reflector being 9 inches.

4. Having given the ratio of the sines of incidence and refraction, when a ray passes out of one medium into each of two others, to find the ratio of the sine of incidence to the sine of refraction out of one of the latter mediums into the other.

5. When parallel rays are incident nearly perpendicularly upon a spherical refracting surface, the distance of the intersection of the refracted ray and the axis, from the centre, is the greatest when the arc is evanescent.

6. A pencil of parallel rays passes from water through a globule of air; find the focus after the second refraction.

7. Find the focal length of a compound lens.

8. Explain the construction of Newton's telescope, and shew how it must be adjusted to the eye of a long-sighted person.

9. An object whose *real* depth below the surface of the water is ten feet, is viewed by an eye fifteen feet above the surface. What must be the focal length of a lens through which it is viewed, that its *apparent* depth may be ten feet?

10. If a plane mirror be turned round uniformly, the angular velocity of the image of a given object formed by continual reflection at its surface : angular velocity of reflector :: 2 : 1.

11. The radii of a spherical reflector and sphere of glass of same aperture and power are in the proportion of 3 : 1. Compare the density of rays in Sun's image formed by them.

12. Two straight lines are inclined at a given angle, and a point E is given without them, a line EFf moves round the point E, and cuts the given lines in F and f; find the locus of the mirror D, so that f shall always be the image of F.

13. Suppose a mirror M to move in a straight line AB, and an object D in the line AC at right angles to it, and distance between object and mirror to be constant; to determine the locus of an eye, which being always at the same distance as the object from the mirror, shall always see the object.

14. Having given the refracting powers of two mediums, to find the ratio of the focal lengths of a convex and concave lens, formed of these substances, which, when united, produce images nearly free from colour.

15. When a ray of light is incident obliquely upon a spherical reflector, to determine the intersection of the reflected ray and the axis of the pencil to which it belongs, and shew that when the focal length is given, the longitudinal aberration of parallel rays varies as (lin. apert.)^2 and lateral aberration varies,as (lin. apert.)^3.

16. If an object be placed in the principal focus of a double convex lens, the visual angle is the same, whatever be the distance of the eye from the glass.

17. Find the length of a caustic generally, and apply it to the case when the reflecting curve is a semicycloid, rays parallel to axis.

18. Find the density of rays in a caustic, when reflecting surface is a hemisphere, radiating point in surface.

TRINITY COLLEGE, 1821.

1. (1). STATE the velocity of light, and the manner in which it was first ascertained.

(2). Prove that the intensity of light, when transmitted through an uniformly dense medium, decreases in geometrical progression.

2. Enumerate the principal discoveries of Newton in the science of Optics; and give a concise account of the experiment by which he determined the refrangibility of light.

3. If an object be placed before a parallel plane mirror, the image will appear to occupy half the space occupied by the object.

4. Two fixed objects situated on the same side of an indefinite straight line, are viewed from different points of that line; required the point at which their apparent distance shall be a maximum.

5. Rays nearly parallel being incident on a spherical reflector whose radius is (r); prove, that if d, d' be the distances of the foci of incident and reflected rays from the surface

$$\frac{1}{d}+\frac{1}{d'}=\pm\frac{2}{r}.$$

according as the reflector is concave or convex.

6. Find an incident ray parallel to the axis of a given reflecting circular arc, that shall be reflected so as to pass through a given point in the circumference.

7. The reflecting curve being the common parabola, and the incident rays perpendicular to the axis, find

(1). The caustic, and

(2). The point where it intersects the axis.

8. If a ray of light be incident obliquely on a small arc (θ) of a spherical reflector, whose radius is (r), and if (d) be the distance of the focus of incidence from the centre; prove that the longitudinal aberration

$$=\frac{2d^2}{(r+2d)^2}\text{ ver. sin.}\theta\text{, very nearly.}$$

9. An infinitely slender pencil of rays is incident nearly perpendicularly upon a spherical refracting surface whose radius is (r); then if (d), (x) denote the distances of the foci of incident and refracted rays from the surface, and if the sine of incidence be (n) times that of refraction,

$$\frac{1}{x} = \frac{1}{r} - \frac{1}{n}\left(\frac{1}{r} - \frac{1}{d}\right).$$

10. (1). If an homogeneous ray of light be refracted through a prism of a denser medium, the deviation will be a minimum, when the refractions on both sides of the prism are equal.

(2). Shew that when this is the case, if (a) be the vertical angle of the prism, (δ) the deviation, and (n) the constant ratio of the sine of incidence to that of refraction.

$$\frac{\sin.\frac{1}{2}(\delta + a)}{\sin.\frac{1}{2}a}.$$

(3). By what methods may the refractive power of any transparent solid substance be determined?

11. Required the focal lengths of a double convex lens of given thickness, the sine of incidence being (n) times that of refraction.

12. Explain the structure of the eye, and the manner in which vision is performed.

13. Shew in what positions of the eye and object, the image formed by double convex lens will be

(1). Magnified or diminished: and

(2). Erect or inverted.

14. (1). Construct Gregory's telescope.

(2). Prove that the field of view depends more upon the aperture of the eye-glass, than of the smaller reflector: and

(3). Shew that the aberration produced by the first reflection, is increased by the second.

15. Given the curvature and aperture of a plano-convex lens, whose plane part is directed towards the object; compare the errors arising from spherical and chromatic aberration.

16. Find at what point in the axis of a given object-glass, a lens of known focal length must be placed, so as to correct the dispersion of an heterogeneous ray refracted through the former

17. Required the curve, in each point of which, the same given brightness, supposed equal to unity, will be produced by a given radiating point.

18. If (F) be the principal focal length of a triple object-glass, composed of three contiguous lenses, whose principal focal distances are p_1, p_2, p_3, respectively; prove that

$$\frac{1}{F} = \frac{1}{p} + \frac{1}{p_1} + \frac{1}{p_2}.$$

TRINITY COLLEGE, 1823.

1. STATE the principal hypotheses which have been formed respecting the nature of light, and the objections which have been raised against each of them.

2. If light, diverging from a luminous point situated in a uniform transparent medium, lose an nth part of its rays in passing through any portion, its intensity after passing through equal successive intervals will decrease as the series

$$\frac{n-1}{n},\ \frac{(n-1)^2}{4n^2},\ \frac{(n-1)^3}{9n^3},\ \frac{(n-1)^4}{16n^4},\ \&c.$$

3. Having given the position of two lights of known intensities, to determine the nature and equation of the surface, of which every point shall be equally illuminated.

4. A reflector is formed by the revolution of a small arc of a catenary round its axis, and rays are incident on it parallel to each other, and to the axis: find the focus of reflected rays.

5. The reflecting curve is the reciprocal spiral, and the radiant point is situated in the pole: find the equation to the caustic.

6. The curvature of the image of a small portion of a sphere placed with its concavity turned towards a concave spherical reflector, is equal to the curvature of the object together with that of a sphere whose radius is the focal length of the reflector.

7. If θ be the angle of incidence of a homogeneal ray of light which passes through a prism, whose refracting angle is α; then when the deviation (δ) is a minimum, $\theta = \frac{\alpha}{2} + \frac{\delta}{2}$.

8. If two parallel rays be incident on a transparent sphere, one perpendicularly, and the other at an angle of incidence $= \theta$; the arc included between the emergent rays is one whose sine

$$= \frac{\sin.2\theta\sqrt{(m^2 - \sin.^2\theta)} - \sin.\theta\sqrt{(m^2 - 2\sin.^2\theta)}}{m^2},$$

$$\text{where } \frac{m}{1} = \frac{\text{sine of incidence}}{\text{sine of refraction}}.$$

9. If F be the focal length of a double convex lens, the radii of whose surfaces are r, r', and its thickness (which is small) $= t$;

$$-\frac{1}{F} = (m-1)\left\{\frac{1}{r} + \frac{1}{r'} + \frac{(m-1)t}{r^2}\right\}, \text{ nearly.}$$

10. Determine the apparent magnitude of a straight rectilinear object, placed at a given depth parallel to a surface of water, the eye being situated in a given point in the plane which passes through the object perpendicular to the surface.

11. Find the form of a surface which shall refract parallel rays accurately to a point within it.

12. If $1 + r$, $1 + v$, be the *ratio of refraction* belonging to the red and violet rays respectively, in a lens whose aperture is D: the diameter of the least circle of aberration is $\frac{v - r}{v + r} \cdot D$.

13. Construct the Gregorian telescope; find its magnifying power, and greatest field of view.

14. Explain the Camera Lucida.

15. If m, n be the *ratio of refraction* belonging to the red and violet rays respectively in a double convex lens, with surfaces of equal radii r; and m', n' those in a double concave lens, and the two be placed so as to coincide; then if the radius of the exterior surface $= \frac{n' - n}{n' - n - 2(m - m')} \cdot r$, the compound lens will be achromatic, the thickness of each being neglected.

16. If the rays of a small pencil of light fall upon a drop of water, and emerge parallel after p reflections within it,

$$\frac{\text{tan. of incidence}}{\text{tan. of refraction}} = p + 1.$$

TRINITY COLLEGE, 1825.

1. PROVE that a ray of light emanating from a given point, and reflected by a plane surface to an eye in a given position, describes the shortest path possible.

2. If rays nearly parallel be incident on a convex or concave spherical reflector, determine the relation between the distances, from the surface, of the foci of incident and reflected rays.

3. A ray of light emanating from a luminous point being incident at an angle $\tan.^{-1}a$, on a curve whose equation is

$$mdx' + ndy' = 0;$$

prove that the equation to the reflected ray is

$$y' - y = \frac{an^2 + 2mn - am^2}{m^2 - 2amn - n^2}(x - x');$$

and thence deduce the general equation to a caustic by reflection.

4. When the reflecting curve is a ⊙, and the radiating point on its $\odot^{ce}$, prove that the caustic is a cardioide, whose base is a ⊙ concentric with the given ⊙, and the radius of which is $\frac{1}{3}$ of the radius of the given ⊙.

5. Find the least ⊙ of aberration, into which a pencil of rays, reflected by a spherical surface, is collected.

6. Give Newton's demonstration of the fundamental law of refraction.

7. Explain the method of determining the refraction

(1). Of a transparent solid,

(2). Of a liquid,

(3). Of a gaseous body.

8. A small pencil of rays being incident on a spherical refracting surface bounding two different media, to find the focus of refracted rays.

9. (1). If F represent the focal length of a double convex lens, the radii of whose surfaces are r and r', and the sine of incidence be (m) times that of refraction, prove that

$$\frac{1}{F} = (m-1)\left(\frac{1}{r} - \frac{1}{r'}\right).$$

(2). How must this formula be modified, so as to exhibit $\frac{1}{F}$ in the meniscus, and in the concavo-convex, and double concave lenses.

10. Prove that the conjugate foci move in the same or in opposite directions, according as the rays are incident on a refracting or a reflecting surface.

11. If a straight line be placed at right angles to the axis of a spherical reflector, or of a lens at a distance (a) from the centre, prove that the polar equation to the image is

$$r = \frac{f}{1 + \frac{f}{a}\cos.\theta};$$

in which r = distance from the centre of any point in the image.

θ = angle subtended at the centre by the straight line,

f = principal focal length of the reflector or lens;

and thence determine the particular cases in which the image is an ellipse, hyperbola or parabola.

12. (1). Construct Newton's telescope, find its magnifying power, and determine its field of view.

(2). What advantage have reflecting telescopes over refractors?

13. If (F) be the focal length of a simple microscope, (c) the nearest distance of vision, then the greatest angle under which the image of a line (a) can be viewed is $a\left(\frac{1}{F} + \frac{1}{c}\right)$.

14. To find the centre and diameter of the least circle of chromatic aberration.

15. Construct an achromatic telescope.

16. Explain the manner in which the primary and secondary bows in the rainbow are formed, and find the altitude of the highest point above the horizon, and the breadth of the colours.

17. Describe the phenomenon of the coloured rings, which are formed by pressing together two plates of glass: and state and examine the hypotheses that have been advanced in explanation of this phenomenon.

TRINITY COLLEGE, 1826.

1. WHAT are the laws of Optics? How are they proved?

2. How does it appear that the propagation of light is gradual?

3. Find the equation between the focal distances for a spherical reflector. Shew that it answers equally for a convex as a concave one.

4. Supposing that a mirror collects the Solar rays to a point at the distance of six inches, where will be the image of an object placed in front of it at twelve feet from its surface?

5. How may the radius of a convex reflector be found by an observation with the Solar rays?

6. Shew that in oblique reflection $\frac{1}{u} + \frac{1}{v} = \frac{4}{\chi}$. ($\chi$ being the chord of curvature through the luminous point.)

7. What is the caustic to a surface generated by the revolution of a cycloid about its axis?

8. What must be the length of a plane vertical mirror, in which a person may see his whole figure reflected?

9. In what case is the image of a straight line produced by a spherical mirror an hyperbola?

10. What are the different species of lenses? Which of them are of the same nature?

11. Find the focal length of a lens, and shew that it is the same whichever side be presented to the incident rays.

12. What kind of glasses are used by persons who have been couched?

13. How are the lenses made to achromatize each other?

14. Shew how a *camera lucida* may be constructed with two small pieces of window glass. What advantage has such an instrument over Dr. Wollaston's?

15. What is the general principle of telescopes?

16. Describe the Dynamometer.

17. Account for the formation of the Rainbow.

18. Describe the phenomena of double refraction.

TRINITY COLLEGE, 1827.

1. The illuminating power of the Sun or Moon, when in the zenith, on a horizontal plane, is to its illuminating power when at any other altitude, as radius to the sine of the altitude.

2. If a ray of light be reflected once by each of two plane surfaces, in such a manner that the planes of the first and second reflections are at right angles to one another; then will the cosine of the inclination of the planes to each other be equal to the product of the cosines of the angles of incidence on each; and the cosine of deviation be equal to the product of the cosines of the doubles of the angles of incidence.

3. When rays are incident on spherical reflecting surfaces, prove that $QF : FE :: FE : Fq$.

4. If y be the semi-aperture of a spherical reflector which is small in comparison with the radius r, then will the longitudinal aberration of parallel rays be $\frac{y^2}{4r}$; and the lateral aberration $\frac{y^3}{2r^2}$.

5. Determine the equation and form of the caustic when the reflecting curve is a circle, and parallel rays are incident in its plane: and shew that the caustic near the cusp approaches indefinitely near a semi-cubical parabola.

6. A ray of light cannot pass out of a denser into a rarer medium, if the angle of incidence exceed a certain limit. Determine this limit in the case of water and air.

7. If a ray of light be refracted through a prism, the deviation will be a minimum when the incident and emergent rays make equal angles with the sides. Apply this property to determine the refracting index of any substance that can be formed into a prism.

8. Find the focal length of the segment of a sphere; both when the parallel rays are incident on the convex surface, and also on the plane.

9. Explain the structure of the Eye.

10. Having given the distance at which a short-sighted person can see distinctly, to find the focal length of a glass which will enable him to see distinctly at any other given distance.

11. To determine the angle which a small object subtends at the centre of the eye when viewed through a convex lens.

12. Explain the construction of the Newtonian Telescope, and shew that objects seen with it appear inverted.

13. Explain the construction of the Magic Lantern; and shew why in practice it is necessary to place a large lens close to the object.

14. Explain what is meant by the dispersing power of any medium; and shew that if a small cylindrical beam of white light pass nearly perpendicularly out of common glass into air, the dispersion of the differently coloured rays is about $\frac{2}{55}$ of the mean refraction.

15. To determine the analytical relation which must hold good in order that two prisms with small refracting angles may form an *achromatic* combination.

16. Find the altitude of the highest point of the rainbow above the horizon, and the breadth of the colours.

TRINITY COLLEGE, MAY 1828.

1. ENUMERATE and explain the phenomena which are observed when a small beam of solar-light is received in different directions on a crystal of Iceland spar.

2. Define the term *deviation*, and prove that if a ray of light be reflected successively at two plane mirrors, so that its whole course be in one plane, the deviation is equal to twice the inclination of the mirrors. Apply this proposition to explain the principle of Hadley's sextant.

3. Define the terms pencil of rays, focus, divergence, convergence, and show how the two last may be measured.

4. **If a pencil of divergent rays be received directly on a concave spherical mirror of principal focal length f, at the distance u from its** origin or focus, the distance v from which it diverges after reflexion is given by the equation

$$\frac{1}{v}=\frac{1}{u}-\frac{1}{f}.$$

5. If the same pencil be received obliquely on the mirror, its axis making an angle ϕ with the normal, the distances v and w, at which the reflected rays meet, (1) in the primary, (2) in the secondary plane, are given by the equations

$$(1).\ \frac{\cos.\phi}{v}=\frac{\cos.\phi}{u}-\frac{1}{f},$$

$$(2).\ \frac{1}{w}=\frac{1}{u}-\frac{\cos.\phi}{f}.$$

6. When a ray passes through a prism, its whole course being in one plane, shew that the deviation has a minimum value given by the equation

$$\sin.\frac{\delta+\iota}{2}=\mu\sin.\frac{\iota}{2}.$$

7. If a percil of rays pass directly through a plane refracting surface at the distance u from its focus, the distance u' of the focus after refraction is given by the equation

$$\frac{1}{u'}=\frac{1}{\mu}\cdot\frac{1}{u}.$$

8. If the incidence be oblique, this equation resolves itself into two as in the case of reflexion:

(1). In the primary plane, $\dfrac{\cos\phi'}{u'}=\dfrac{1}{\mu}\cdot\dfrac{\cos.\phi}{\cos.\phi'}\cdot\dfrac{\cos.\phi}{u}.$

(2). In the secondary plane, $\dfrac{1}{u'}=\dfrac{1}{\mu}\cdot\dfrac{1}{u}.$

9. When a pencil of rays passes through a convex refracting surface, diverging both before and after refraction, if the incidence be direct

$$\frac{1}{u'}=\frac{1}{\mu}\cdot\frac{1}{u}-\frac{\mu-1}{\mu}\cdot\frac{1}{r}.$$

If the incidence be oblique the equations are

$$(1).\quad \frac{\cos.\phi'}{u'} = \frac{1}{\mu}\cdot\frac{\cos.\phi}{\cos.\phi'}\cdot\frac{\cos.\phi}{u} - \frac{\mu\dfrac{\cos.\phi'}{\cos.\phi} - 1}{\mu\dfrac{\cos.\phi'}{\cos.\phi}}\cdot\frac{1}{r},$$

$$(2).\quad \frac{1}{u'} = \frac{1}{\mu}\cdot\frac{1}{u} - \frac{\mu\dfrac{\cos.\phi'}{\cos.\phi} - 1}{\mu}\cdot\frac{\cos.\phi}{r}.$$

10. When a pencil of rays passes through a thin double convex lens, diverging after emergence, if the axes of the pencil and lens coincide,

$$\frac{1}{v} = \frac{1}{u} - (\mu - 1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right).$$

If these axes make with one another an angle ϕ, the pencil passing through the centre of the lens,

$$\frac{\cos.\phi}{v} = \frac{\cos.\phi}{u} - \left(\mu\frac{\cos.\phi'}{\cos.\phi} - 1\right)\left(\frac{1}{r_1} + \frac{1}{r_2}\right),$$

$$\frac{1}{w} = \frac{1}{u} - \left(\mu\frac{\cos.\phi'}{\cos.\phi} - 1\right)\left(\frac{1}{r_1} + \frac{1}{r_2}\right)\cos.\phi.$$

11. These last formulæ apply also to the case of a pencil passing through any part of a lens, provided its axis make equal angles with the surface at incidence and emergence.

12. Find the equation to the curve which by its revolution round the axis of x will produce a surface capable of refracting a large pencil of rays accurately from one focus to another, and show that a spherical surface may be found which will answer this condition.

13. Show from the formulæ, that when a pencil of rays falls directly on a concave mirror or a convex lens, it receives a certain quantity of convergence depending merely on the curvature of the surfaces, and that the same proposition holds in reflection with respect to oblique pencils, in the primary but not in the secondary plane.

14. Show in like manner that when a divergent pencil passes directly from a rarer into a denser medium, its divergence is diminished in a given ratio depending merely on the relative refracting powers of the media, and that if the surface separating the media

be convex towards the rarer one, the pencil after being thus modified receives a certain quantity of convergence depending on the ratio of refraction and the curvature.

15. When a convex lens of focal length f is used without any limitation of the incident pencils to produce an image of a plane object, the rays of each pencil converge to different points included between two surfaces which touch one another, having for their radii of curvature at the common vertex

$$\frac{\mu}{3\mu+1}f, \text{ and } \frac{\mu}{\mu+1}f.$$

16. Explain the effect of the chromatic dispersion of light in the case (1) of the object glass of an astronomical telescope, (2) of the single eye-glass. Explain also the manner of correcting the defect in each case.

17. Explain the construction of Galileo's telescope, and point out all the particulars in which it differs from other telescopes.

18. Calculate the magnifying power and field of view of a Gregorian telescope from the following data:

Object mirror, focal length	12	inches	
Small mirror	1	....	
Field glass	3	....	distance 2 inches
Eye glass.....................	1	....	
Field-bar aperture	$\frac{5}{8}$	inch.	

19. The common formula for the lens, in which the thickness is neglected, is accurately true if the distances represented by u and v be measured from a point within the lens, the distances of which from the surfaces of the lens are inversely as the refractions at those surfaces.

20. Account for the formation of the ordinary rainbow, and for the brightness of that portion of the sky which lies within the bow.

TRINITY COLLEGE, JUNE 1829.

1. WHAT hypotheses have been made to explain the phenomena of Light? To what objections are they respectively liable?

2. Assuming that rays of light may be represented by straight lines, what is meant by the divergence of a pencil of rays?

3. Investigate the ordinary simple formula for reflexion at a spherical surface, and show that its import is: that the divergence destroyed, and the convergence produced, constitute together a given quantity of effect.

4. Exhibit the aberration of a spherical mirror for parallel rays, and show that it is nearly equal to half the distance of the point where the reflexion takes place from a plane touching the surface at the place of direct incidence.

5. Prove that two successive reflexions, at plane mirrors inclined to each other, produce a deviation double of the angle of inclination of the mirrors.

6. Determine the greatest possible number of reflexions that a pencil can suffer at two plane surfaces inclined to each other at a given angle.

7. Explain the law of ordinary refraction.

8. Investigate the formula for direct refraction at a spherical surface,

$$\frac{\mu}{u'} = \frac{1}{u} - \frac{\mu - 1}{r} - \frac{\mu - 1}{2\mu^2}\left(\frac{1}{r} + \frac{\mu + 1}{u}\right)\left(\frac{1}{r} + \frac{1}{u}\right)^2 y^2,$$

first without its last term (explaining its import): and secondly with the last term.

9. Investigate the ordinary formula for a double convex lens.

10. Find the aberration of a double convex lens, and show that for parallel rays it is least when the radii are to one another as 1 : 6.

11. Explain what is meant by the primary and secondary planes in the case of an oblique pencil, and find the places of the lines of convergence in oblique reflexion at a spherical mirror.

12. "The best form for the ground, or bottom, of a camera obscura, is that of a sphere of which the radius is three eighths of the focal distance, when a double convex lens of crown glass is employed." Dr. Young's *Lectures*, Vol. I. p. 425. Demonstrate this proposition, and explain Professor Airy's construction by which a perfectly distinct image is produced.

13. Explain the construction of the astronomical telescope, with the positive and negative eye-pieces. Why does the latter give the Moon a spherical appearance?

14. The focal length of the object-glass being given, and the eye-piece being a single lens, show upon what depend: (1) the magnifying power, (2) the field of view, (3) the quantity of light concentrated in each point of the image.

15. The focal lengths of the lenses in a four-glass eye-piece are 3, 4, 4, and 3 inches respectively, and the intervals between them, reckoning from the object-glass, 4, 6, and $5\frac{1}{3}$. Required the focal length of the equivalent single lens.

16. Account for the laws of reflexion and refraction of parallel rays of light on the theory of Undulations.

17. Explain the phænomenon of the black stripes produced by the union of two streams of light in Fraunhofer's experiments.

18. Prove that a small object can be seen distinctly through a prism only when each pencil makes nearly equal angles with the surfaces at incidence and emergence, and that it then appears partly magnified and partly contracted in one dimension, but not in the transverse one.

TRINITY COLLEGE, May 1830.

1. Define a *ray*, and a *pencil* of rays, and show that the theory of Undulations necessarily allows of the same mode of representing these, as that of Emanation, in all questions relating to reflexion and refraction.

2. A pencil of parallel rays falls directly on a concave spherical mirror: required its form after reflexion.

3. A pencil of rays falls obliquely but centrically on a concave mirror: explain the positions of the *primary* and *secondary* planes,

and determine the points at which the rays in those planes are collected.

4. Determine the form and position of a reflecting surface which shall collect accurately to one point, rays which have proceeded from a given point.

5. A very distant point is seen by rays successively reflected by two plane mirrors given in position: required the angle of deviation between its real and apparent directions.

6. Prove that the deviation caused by a prism of a small refracting angle is nearly expressed by $(\mu - 1)\iota$, where ι is the refracting angle, but that it is in all cases greater than this quantity.

7. The radii of the surfaces of a double convex lens being represented by r and s, determine the principal focal length.

8. Determine the lens of least aberration for parallel rays.

9. Prove that the divergence of a pencil of rays is diminished, in its passage directly through a convex lens, by a quantity depending only on the curvatures of the surfaces, and the refracting power of the substance.

10. Assuming

$$V = \frac{1}{3(\alpha - \beta)^2}\{28x^2 + 20(\alpha + \beta)x + 10\alpha\beta + 3\beta^2 + 27\},$$

determine the form of a lens which shall produce a distinct image of a distant object of large extent, the pencils being defined by a diaphragm placed behind the lens at one-sixth of the focal length from it.

11. Find the principal focal length of a sphere, and the aberration for parallel rays.

12. Determine the form of the image produced by a sphere, with centrical pencils, the surface of the object being plane.

13. Explain what is meant by irrationality, the chromatic dispersion.

14. Explain the construction of the Opera-glass.

15. The focal length of the object-glass of an Astronomical Telescope, is 5 feet; the eye-piece consists of two lenses of focal lengths 1½ inch and ½ inch: determine the position of these when the instrument is in full adjustment.

16. Explain the principle of the simple Microscope, shewing on what depend the magnifying power, and field of view.

17. The focal length of the field-glass of a Microscope is 1 inch, that of the eye-glass $\frac{1}{2}$ inch: the distance of the field-glass from the centre of the object-glass is 2 inches: determine the interval between the lenses when the vision is achromatic, and shew that the bending of the pencil is equally divided between the two lenses.

18. Explain the construction of the Camera Obscura. Shew how the result obtained in the answer to Question 10 may be applied, combining the refractions with internal reflection, as in Chevalier's "Chambre obscure à prisme menisque."

19. Account for the law of refraction on the hypothesis of Undulations.

20. Explain the phænomenon of the dark bands produced by the united effects of two streams of light in directions nearly coinciding.

St. JOHN'S COLLEGE, Dec. 1820.

1. Define the terms geometrical focus and principal focus; and find the focal length of a sphere placed in a denser medium.

2. Two lights whose intensities are as $a : b$ are placed in the foci of a given ellipse. Find the point where the least light is received; and shew, that if from any point a normal be drawn, proportional quantities of light are received at its extremities.

3. Explain the formation of the primary rainbow, and why it cannot be greater than a semicircle.

4. If R and r be the radii of the surfaces of a glass lens, whose thickness is inconsiderable, and which is connected with another thin glass lens, the radii of whose surfaces are R' and r', prove that the focal length of the compound lens may be found from the equation

$$\frac{1}{\text{focal length}} = \frac{1}{2}\left(\frac{R \overset{+}{\sim} r}{Rr} \overset{+}{\sim} \frac{R' \overset{+}{\sim} r'}{R'r'}\right).$$

5. Having given the radii of a thin double concave lens, upon which parallel rays are incident, find the radii of an isosceles double convex lens which compounded with the former shall refract the rays parallel.

6. If an object be placed between two parallel plane reflectors, which are moved parallel to themselves, their distance remaining constant, shew that the images formed by an even number of reflections will remain stationary, and that the other images will move in the same direction as the reflectors, and with twice their velocity.

7. Construct a Gregorian telescope, and determine the field of view.

8. $ABCDEF$ is a regular hexagon inscribed in a circle whose centre is G: GA, GB are two plane reflectors, AB is an object placed between them. Prove that the images of AB will form the remainder of the hexagon; and shew whether an eye placed between the two reflectors, sees one, or both, or part of each of the two images which coincide in DE.

9. If an object be placed before a double convex lens, find the position of the eye that the object may always appear of the same magnitude wherever it is placed.

10. A stick of given length (l) is placed within a polished hemisphere of given radius (r). Find the dimensions of the image; and shew that it will be an ellipse, hyperbola, or parabola, according as $\frac{l}{r}$ is less, greater, or equal to $\sqrt{3}$.

11. In a parabolic reflector, a ray of light diverges from a point Q in the axis, and is reflected at P so as to cut the axis in q. Shew that the distance between q and the geometrical focus

$$= \frac{AS^2}{SQ}\left(\sec.^4 \frac{ASP}{2} - 1\right).$$

12. The reflecting curve is an arc of 60° and the focus of incident rays at one extremity. Find the length of the caustic, trisect it geometrically, and compare the densities at the points of trisection.

ST. JOHN'S COLLEGE, DEC. 1824.

1. REQUIRED the least breadth of a vertical plane mirror, in which a man six feet in height may see his own image complete; and its height from the ground, supposing his eye to be four inches below the crown of his head.

2. A ray of light cannot pass out of a denser into a rarer medium, when the angle of incidence exceeds a certain limit when sin. inc. : sin. ref. : : 1 : 2.

3. S and H are fixed points; a ray SP proceeding from S, is reflected to H by a plane mirror AP moveable above a given point A in the line SH produced; required the curve, which is the locus of P.

4. Given the distance (a), to which a short-sighted person can see distinctly; find at what distance from his eye a concave lens of given focal length (v), must be placed, to enable him to see an object Q at a distance (b); and explain what is to be understood by that result, which places the lens beyond Q.

5. The whole circumference of a given circle is luminous, except a very small arc, which reflects the light from the vertical. Required the form of the caustic.

6. Sin. inc. : sin. ref. : : m : 1; rays diverging from a given point Q, enter nearly perpendicularly into a sphere, the opposite surface of which is quick-silvered. Find the geometrical focus of emergent rays.

7. The image of a straight line formed by a single spherical refracting surface is a conic section; shew that if the line be placed within a sphere of glass, the image must be an hyperbola. And explain how we may suppose it to be completed.

8. If Q be the focus of rays incident on a spherical refracting surface, and Q' the focus of incident rays coming in the contrary direction, such that Q and Q' have the same conjugate focus, required a maximum and minimum value of $Q\,Q'$.

9. There is a small aperture of the shape of an isosceles double convex lens of inconsiderable thickness, in the centre of a glass sphere; if this aperture be filled with a given substance, find the focal length of the sphere in air. Supposing m and m' to be the values of $\frac{\text{sin. inc.}}{\text{sin. ref.}}$ out of air into glass, and the interior substance respectively.

10. If parallel rays be incident in a curve in a direction perpendicular to the axis of x; prove that the co-ordinates x', y', to the point where the reflected ray meets the caustic, are determined by the

equations $x' - x = -\frac{p}{q}$, $y' - y = -\frac{p^2}{2q}$, where $p = \frac{dy}{dx}$, $q = \frac{d^2y}{dx^2}$, and hence shew that the caustic to the logarithmic curve

$$x = a \,.\, \text{hyp. log.} \frac{y}{a},$$

is the common catenary.

11. Find the longitudinal aberration when a pencil of parallel rays is incident on a spherical refracting surface; and if (θ) be the angle at which the extreme ray is incident, and $\frac{\text{sin. inc.}}{\text{sin. ref.}} = m$, shew that the lateral aberration = rad. $\sin.\theta$ vers. $\frac{\theta}{m}$ nearly.

12. Construct Cassegrain's telescope, and find its field of view, also compare its magnifying power with that of a Newtonian telescope, having the same eye-glass, and large reflector.

ST. JOHN'S COLLEGE, MAY 1826.

1. EXPLAIN the construction of the single microscope, and find its magnifying power.

2. At what point in the axis of an elliptical reflector must Q be placed that q may be at the extremity of the axis major.

3. A glass sphere is divided into 3 parts by two parallel planes which trisect its diameter at right angles. The glass between the planes is removed, and its place supplied by water. Find the distance between the images of two objects placed in the centre, and at the extremity of the diameter.

4. Q is an object placed between two inclined plane reflectors. Find the locus of an eye such that the path of the ray (from Q to the eye) by which the nth image of one side of Q is seen may be of the same length as that by which the nth image of the other side of Q is seen.

5. Of how many degrees must a polished circular arc be, that the image of its tangent may be a quadrant of an ellipse?

6. A straight line is inclined at a given angle to the side of a glass prism whose vertical angle is known. Find the nature and position of the image.

7. If a Gregorian telescope be adjusted to the eye of a short-sighted person, and the eye be farther from the lens than the principal focus, what change is produced in the visual angle, and in the position of the image on the retina?

8. Find the thickness of a plane glass mirror that the distance of the image from the first surface may be twice as great as in a mirror of inconsiderable thickness.

9. Investigate the nature of the reflector that parallel rays incident upon it in the same plane may be reflected parallel.

10. When diverging rays are refracted by a medium contained by parallel plane surfaces, the aberration is less than when they are refracted by a single surface.

11. Find the form of the surface of a medium such that rays proceeding from a point without it may be refracted accurately to a given point within it.

ST. JOHN'S COLLEGE, MAY 1827.

1. If a man 6 feet high be placed before a vertical mirror nearly his own height, at a distance of 4 feet, required how much of his person will be illuminated by a lamp behind him, 12 feet from the mirror and 11 feet from the ground; and find his position when his feet are just illuminated.

2. A ray of light suffers as much deviation in passing through one medium contained by parallel plaue surface into another, as in passing immediately into the latter medium.

3. Given the focal length of a lens, which will just enable a short-sighted person to receive parallel rays, to determine the focal length of one, which will enable him to see to any given distance.

4. Rays tending to form an image at the point where the eye is placed, are received upon a concave lens, required to prove that the visual angle varies inversely as the square of the distance of the lens from the eye.

5. Given the focal length of a glass lens in air, find another, which, being compound with it, and the whole placed in water, the focal length shall equal that of the first in air.

6. When rays are incident parallel to the axis of a cycloid, to determine the form of the caustic and the law of the density.

7. Rays diverge from a point in the axis of double convex lens whose thickness equals one of its radii. Required the geometrical focus of refracted rays.

8. The image of a straight line perpendicular to the axis of a concave spherical reflector, and passing through its centre as seen by an eye placed in the axis at such a distance that all lines drawn to it from the reflector may be considered parallel, is determined by the equation

$$y\,.\,(r^2 + 2x^2) = 2x^2\sqrt{(r^2 - x^2)}.$$

Investigate this, and trace the curve.

9. In the last question, find in what part of the reflector the image of the diameter is formed. And explain clearly where the eye must be supposed to be placed when the image of a straight line formed by a spherical reflector is said to be a conic section.

10. An homogeneous pencil of parallel rays is incident on a sphere of refracting substance, so as to emerge parallel after (p) internal reflections; required the angle between the incident and emergent pencils.

11. The lunar apertures of the object and eye-glass of an astronomical telescope being as 6 : 1, and their focal lengths 27 inches, and $\frac{1}{2}$ inches respectively, required the focal length of a convex lens to be placed between them one inch from the eye-glass, and of twice its aperture, that the field of view may be doubled, when adapted to common eyes. Also determine the effect upon the magnifying power of the telescope.

12. Compare the focal lengths of two lenses of different given substances, which when compounded shall cause the red and violet rays of a pencil to converge to the same focus; and shew that if the spectrum formed by similar prisms of each substance were divided exactly in the same proportion by the different colours, the compound lens would be wholly free from colour.

St. JOHN'S COLLEGE, May 1828.

1. Explain by what contrivances in the eye objects are seen distinctly at different distances and with different degrees of light.

2. Given Q the focus of rays incident nearly perpendicularly on a concave reflecting sphere; find q the focus after two reflections.

3. Find by experiment the principal focus of a lens, or spherical reflector.

4. Archimedes' burning mirror is supposed to have been formed by small planes reflecting the Sun's rays to the same point. Find the surface they all touch, and explain what limits its effects.

5. Given the interior, find the exterior radius of a meniscus by which a person who can see distinctly at one given distance, may be enabled to see distinctly at another, and explain the advantage of this form of lens for spectacles.

6. If $\sin. I : \sin. R :: \sqrt{3} : 2$, out of the air into a sphere, find what part of the Sun's rays incident on the sphere are transmitted.

7. Given Q the focus of rays incident nearly perpendicularly on a prism AIC, find q the focus of refracted rays, and also the curve described by q if the prism increase in thickness by the revolution of the side IC round I.

8. Compare the magnifying powers of a sphere and a plano-convex lens of the same radius, and shew by what contrivance the two may be made equal, and the advantages of a plano-convex lens so formed.

9. Given the diameters and focal lengths of the object and eye-glass in the Astronomical telescope; find the area of the bright part of the field of view, and explain the use of the stops and additional glasses usually found in telescopes.

10. In Cassegrain's telescope the spherical aberrations partly correct each other, but cannot be made to do so entirely.

11. If the dispersive powers of two media are as $3 : 2$, find the focal lengths of two lenses which will, combined, produce an achromatic lens of 12 inches focus.

12. Find the caustic of rays diverging from a luminous point, and reflected by a plane silvered mirror.

St. JOHN'S COLLEGE, June 1829.

1. When diverging or converging rays are incident nearly perpendicularly upon a spherical reflector, the conjugate foci lie on the same side of the principal focus.

2. The concavity of a meniscus of glass of inconsiderable thickness is filled with water; the radii of the surfaces are 5 and 6 inches: required the focal length of the compound lens.

3. The surfaces of a concavo-convex lens of inconsiderable thickness are formed by the revolution of an ellipse round its major and minor axes. Given the focal length of the lens and the axes of the ellipse, determine the refracting power of the medium.

4. In the astronomical telescope, the linear magnitudes of the visible area and the bright part of the visible area are

$$\frac{D}{F} - \frac{D-d}{F+f}, \text{ and } \frac{D+d}{F+f} - \frac{D}{F};$$

where D, F and d, f are the diameters and focal lengths of the object and eye glasses.

5. A ray of light parallel to the axis of a paraboloid of glass after refraction cuts the axis in a given point; required the point of incidence.

6. A fire is placed in the axis of a concave mirror, which reflects one-eighth part of the heat, at a greater distance from it than the principal focus. Where must a small object be placed in the axis between the fire and its geometrical focus that it may be the least heated?

7. Two lights are placed in the opposite extremities of the diameter of a circle, the intensities of which are :: 4 : 3. What part of the curve is the most illuminated?

8. What spherical reflectors and lenses from aberration are more powerful in the centre than at a distance from it, and the converse; and how are the effects of this imperfection partially counteracted in the telescopes of Galileo and Cassegrain?

9. If the *cosines* of incidence and refraction be to one another :: $m : 1$, the surface, which will refract parallel rays accurately to a point S, is formed by the revolution of the curve $r = a\dfrac{\left(\tan.\frac{\theta}{2}\right)^m}{\sin.\theta}$ round an axis drawn through S in the direction of the incident rays; where r is the distance of any point from S, θ the inclination of r to the axis, and a an arbitrary value of r when $\theta = 90°$.

10. A luminous body is surrounded by a medium which absorbs light with a power varying inversely as the distance from it. At the distances 1 and s the intensities of light are s^m and s^n; prove that at the distance r the intensity is $s^m . r^{n-m}$.

11. Prove that the altitude of the primary rainbow $= 4R - 2I - S$; where $S =$ the Sun's altitude, $I = \cos.^{-1}\sqrt{\dfrac{r^2 - 1}{3}}$, and $r = \dfrac{\sin.I}{\sin.R}$, for red rays. What is the corresponding formula for the secondary rainbow?

12. When a small pencil of homogeneal rays falls *obliquely* upon a spherical refractor, in a plane which passes through its centre, having given the focus of incident rays and the angles of incidence and refraction, required a formula to determine the geometrical focus of refracted rays.

ST. JOHN'S COLLEGE, MAY 1830.

1. COMPARE the illuminations of a horizontal and an inclined plane, the sky being covered with clouds of uniform brightness.

2. Having given the breadth of a man's face, and the distance between his eyes: find the breadth of the narrowest plane mirror in which he can see the whole of his face.

3. Find the successive images of a point placed between two concentric concave reflectors, of equal focal length.

4. Find the nature of the caustic, when the reflecting surface is a spheroid, and the rays are incident parallel to its axis.

5. Compare the real and apparent diameters of the bore of a barometer tube.

6. When a ray of light passes in *any* direction through a prism, the incident and emergent rays make equal angles with the edge of the prism.

7. Find the angle subtended by a small object viewed through a lens not placed close to the eye.

8. In the above question find the distance of the object from the lens, the focal length of the lens being F, its distance from the eye k, and the distance of most distinct vision c.

9. Find the radii of the surfaces of a lens of given focal length, when the longitudinal aberration for parallel rays is a minimum, the value of μ being 1·5.

10. When a pencil of homogeneal rays is refracted into a sphere in such a manner that the rays emerge parallel after two internal reflections, determine the ultimate intersections of the rays in the plane of incidence. Determine also their ultimate intersection (after the first refraction) in a plane perpendicular to the plane of incidence.

11. When a beam of white light is incident on a prism in a plane perpendicular to its edge, the angle through which the emergent pencil is dispersed,

$$= \sin. I \,.\, \text{secant}\, \phi_1 \;\, \text{secant}\, \psi \,.\, \delta\mu \text{ nearly.}$$

12. Describe the Magic Lantern. If instead of having one lens between the picture and screen, it has two of equal focal lengths $3c$ placed at the distance $2c$ from each other, find the least distance between the picture and screen.

13. The focal lengths of the object-glass, field-glass, and eye-glass of a telescope, are $60c$, $6c$, and $2c$ respectively, the diameter of the object-glass is $3c$, that of the eye-glass is c: find the power of the telescope, and compute roughly the diameter of the field-glass when just large enough not to contract the field of view.

QUEEN'S COLLEGE, 1824.

1. A GIVEN vertical object is supposed to be a given height above an horizontal plane, required the point in the plane from which the object will appear to have the greatest magnitude.

2. Two points and an indefinite straight line placed between them, not at right angles to a line joining the points, being given in position; find a portion of the line such, that its apparent magnitude seen from one of the points may be double of the apparent magnitude seen from the other.

3. If a plane mirror revolve about an axis, the angular motion of the image of any object is double of the angular motion of the mirror.

4. A radiant point and the position of the eye being given, both on the same side of a plane mirror; if the mirror be supposed to move in a direction perpendicular to its own plane, required the locus of the several points of the mirror from which rays are reflected to the eye.

5. It is required to find an incident ray parallel to the axis of a given reflecting circular arc, which shall be reflected so as to pass through a given point in the axis.

6. If a ray parallel to the axis of a glass sphere, one half of which is covered with a reflecting substance, fall upon the refracting half of the sphere; after refraction it shall be reflected to a point in the axis, at a distance from the reflecting surface nearly equal to one sixth part of the sphere's diameter.

7. Having given the distance of an image from a double concave lens, and the ratio of the object to the image; required its focal length.

8. How many radii of the convex surface of a plane convex lens must an object be placed from it, so that its magnitude may be (m) times greater than that of its inverted image.

9. An object viewed with a Gregorian telescope is magnified (m) times the focal length of the eye-glass is (a) inches, and the distance between their principal foci given; required the relation of the focal lengths of the two reflectors.

10. If the thickness of two microscopic glasses be $\frac{1}{8}$ of their radii of convexity, and these be in the ratio of 10 to 3, how must the glasses be disposed in a double microscope, so that an object 8 inches distant from the eye may be magnified 1000 times.

11. When parallel rays are incident upon a spherical reflector, show that the radius of the least circle of aberration varies directly as the cube of the semi-aperture, and inversely, as the square of the focal length of the reflector.

12. If the Sun be on the horizon, what is the altitude of an hill from the top of which a rainbow will appear to be perfectly circular?

13. The radiating point and the caustic being given, show that there are an infinite number of reflecting curves which will produce the caustic.

QUEEN'S COLLEGE, 1827.

1. Place an object before a double convex lens, so that the image may be double of the object, and erect.

2. Determine the apparent magnitude of a straight line, placed at a given depth, parallel to the surface of a vessel of water, the eye being situated in a given point in the plane passing through the object perpendicular to the surface.

3. Two plane reflectors are inclined to each other at an angle of 30°: required the number of images of an object situated between them.

4. When the reflecting curve is a cycloid, and the rays proceed parallel to the axis, find the caustic.

5. Find the magnifying power of a simple microscope.

6. Explain the Camera Lucida.

7. In a double convex lens, find the position of the conjugate foci when their distance from each other is a minimum.

QUEEN'S COLLEGE, 1828.

1. The foci of incidence and refraction of rays falling nearly perpendicularly on a concave mirror of given radius, are on opposite sides of the mirror, and the focus of refraction twice as far from it as the focus of incidence; find their actual distances.

2. A pencil of parallel rays is incident nearly perpendicularly on a plano-concave lens of given thickness, and after reflexion at the plane side, which is silvered, is again refracted out of the lens; find the focus of emergent rays.

3. A sphere of air is included in a medium of glass through which parallel rays are passing. What portion of the rays incident on the sphere will pass through it?

4. Find the nature, length, and law of density of the caustic when the reflecting curve is a circular arc, and the focus of incident rays is in the circumference.

5. Given the radius of the arc of any colour in the primary rainbow: find the refraction when rays of that colour pass out of air into water.

6. At what altitude above the centre of a circular annulus must a luminous point be placed, that the quantity of light on the annulus may be the greatest possible?

QUEEN'S COLLEGE, May 1829.

1. Find the direction of a ray passing through a given point, so that after being reflected at two plane mirrors, given in position, it may pass through the same point again.

2. AC is vertical, CB horizontal, AB a straight line of given length, which reflects all rays incident upon it from a luminous point Q, placed at a given distance from C in CB produced. Find the inclination of AB that half the rays incident upon it may, after reflection, meet CA produced.

3. A luminous point is placed in the axis of a concave mirror of one foot radius, at the distance of three feet from it: to what point

are the rays reflected? also, determine the ratio of the initial velocities of the foci of incidence and reflection in this case.

4. A luminous point is placed in the centre of gravity of a right cone, of a given altitude of a denser medium; and its vertical angle is as large as possible, that no rays incident on the base may be reflected. Required the content of the cone.

5. Find the nature, length, and variation of density, of the caustic, when the reflecting curve is a circle, and the focus of incidence is in the circumference.

6. Find the focal length of a double convex lens of equal radii and inconsiderable thickness: and then, supposing it to be double of either radius, determine the refractive power of the medium.

7. Construct Newton's telescope; shew that objects appear inverted through it; find the field of view; and trace the course of a pencil of rays on supposition that a rectangular prism is substituted for the plane reflector.

8. A luminous point is in the middle of the axis of a hollow cylinder of given length, and just half the rays diverging from it fall on the interior surface. From these data, compare the intensity of the light at the edge, with that in the middle of the surface.

QUEEN'S COLLEGE, MAY 1830.

1. A LUMINOUS point is placed at a given distance in the perpendicular through the centre of a given square: find the whole light incident on the square.

2. A concave spherical reflecting surface is filled like a cup with water; given the focus of rays diverging from a point above the surface of the water, to find the focus of rays which emerge after having suffered two refractions and one reflection.

3. A luminous point Q, is placed at four feet distance, in the axis of a convex mirror of one foot radius; find the focus q of reflected rays: also find the ratio of their initial velocities, supposing Q to be put in motion.

4. A pencil of rays diverging from the centre of a sphere, after refraction at its surface, diverge from the opposite extremity of the diameter; required the refractive index.

5. A telescope consists of three convex lenses, whose focal lengths are as 30, 3, and 1; the latter two being at the distance 2 from each other: find the magnifying power, and trace the course of the rays.

6. On supposition of the Sun's light being homogeneous, shew what would be the nature and width of the primary rainbow; and determine from observations made on such a bow, the refrangibility of the light in passing out of air into water.

7. Prove that the deviation of a ray which passes through a prism in a plane perpendicular to its axis is a minimum when the incident and emergent rays make equal angles with the sides of the prism: and shew that there could be no such minimum, if the *angles* of incidence and refraction, instead of their *sines*, were in a constant ratio.

CORPUS CHRISTI COLLEGE, 1825.

1. What are the two principal hypotheses which have been formed respecting the nature of Light? How has it been found that the action of light is not instantaneous? What are the three laws on which the theory of Optics is founded, and how are they established by experiment?

2. Q and q are the conjugate foci of a small pencil of parallel rays, incident nearly perpendicularly on a spherical reflector, F is the principal focus, $QE=q$, $Eq=q'$, $EF=f$; shew that $\frac{1}{q'}=\frac{1}{f}+\frac{1}{q}$; and hence prove that Q and q lie on the same side of F, that they move in opposite directions, and meet at the centre and surface of the reflector.

3. Determine the aberration in reflection, at a spherical surface.

4. Parallel rays are incident on a parabola in a direction perpendicular to its axis; find the equation to the caustic by reflection, and the angle at which it cuts the axis of the parabola.

5. PR is a straight line placed before a spherical reflector, and inclined at a given angle to EP. Prove that its image is a conic section. Determine the angle ϕ which the axis-major makes with EP at E, and shew that the image is an arc of an ellipse, hyperbola,

or parabola, according as $EP \cos\phi$ is greater than, less than, or equal to, one half the radius of the reflector.

6. Determine the deviation of a ray of light refracted through a triangular glass prism, and shew that it is a minimum, when the incident and emergent rays make equal angles with the sides of the prism.

7. If a plane reflector turn round an axis, shew that the angular motion of an image formed by reflection, is double that of the reflector.

8. Δ and Δ' are the distances of the foci of incident and refracted rays from the surface of a spherical refractor, whose radius $= r$, shew that

$$\frac{1}{\Delta'} = \frac{m-1}{mr} + \frac{1}{m\Delta},$$

the direction of incidence being nearly perpendicular to the surface, and deduce the values of Δ' for rays incident on the concave and convex surfaces, both of a rarer and a denser medium.

9. $ABab$ is a sphere whose radius $= r$, Bcb is a spherical surface whose radius $= r'$, $BAbc$ is glass, and $Babc$ any other substance whose refracting power is given; if QA be a ray of light refracted through the sphere, find the geometrical focus of refracted rays.

10. Find the aberration in a lens whose thickness is inconsiderable.

11. If $1 + r$, $1 + v$, be the ratio of refraction for red and violet rays respectively, in a lens whose aperture $= a$, shew that the diameter of the least circle of chromatic aberration $= \dfrac{v-r}{v+r} \cdot a$.

12. AB is a lens whose focal length $= a$; suppose that a short-sighted person, unable to see distinctly objects beyond distance a, is also unable to see those within the distance $\dfrac{a}{m}$; find the least value of Δ, for which the above lens will be of use.

13. Construct the Gregorian telescope, and find its magnifying power.

14. Given the distance at which a short-sighted person can see distinctly, find the distance between a given object-glass, and a given

eye-glass in the astronomical telescope, when adapted to such an eye, and to distant objects.

15. Determine the focus of a thin pencil of rays, after being refracted obliquely at a curve surface; and shew that if the surface be plane,

$$v : u :: \frac{\sin.\phi}{\cos.^2\phi} : \frac{\sin.\rho}{\cos.^2\rho}, \text{ where } QP = u,\ Pq = v;$$

ϕ = angle of incidence, and ρ = angle of refraction.

16. The colours of the secondary rainbow occur in the inverse order of those in the primary.

17. Find the altitude of the highest point of the rainbow above the horizon, and the breadth of the colours.

CORPUS CHRISTI COLLEGE, June 1829.

1. Explain the properties of light upon which the laws of Optics depend. State how and by whom it was discovered that the propagation of light is not instantaneous, and describe accurately the experiment by which Newton determined that it was composed of rays differing in refrangibility and colour.

2. An object is placed between two plane mirrors not parallel. Find the number of images which will be formed of it by successive reflections, and apply the principle to explain the construction of the Kaleidoscope.

3. If a pencil of divergent rays be received upon any curved reflector at a distance u from its focus, the distance v of their intersection after reflection will be found from the equation $\frac{1}{u} + \frac{1}{v} = \frac{4}{c}$, c being the chord of curvature passing through the focus of incident rays.

4. Determine the caustic by reflection at a spherical surface after two reflections of the incident rays, the luminous point being in any part of the surface.

5. Give a practical method of determining the refractive power

of any substance, solid or fluid; and find the angle at which a ray must be incident upon a prism so that the deviation in passing through the prism may be a minimum, the whole course of the ray being in a plane perpendicular to the axis of the prism.

6. Find the geometrical focus of rays after refraction at a spherical surface of a refracting medium: and determine that focus of incidence which will have its geometrical focus of refracted rays at the least distance from it: also that focus of incidence for which there will be no aberration.

7. Find the principal focus of a sphere, and apply the expression to the case of a sphere of glass placed in a medium of water, the luminous point being within the medium.

8. Define the centre and focal centres of a lens, and determine the latter for the meniscus.

9. Parallel rays are incident upon a double convex lens of which the second surface is silvered; the radii of the lens are 5 and 7 inches respectively, and the index of refraction 2·45. Find the focus of emergent rays.

10. Determine the spherical aberration of a lens, and shew that it varies as the square of the radius of the aperture.

11. Explain fully the construction of the human eye, and shew to what optical instrument it is very nearly analogous.

12. Determine the diameter of the least circle of chromatic aberration.

13. Define dispersive power, and construct an achromatic prism.

14. Prove that rays which after reflection or refraction tend to form an image, may be refracted to the eye by a double concave lens, so as to form a distinct image on the retina: and determine the variation of the visual angle in this lens.

15. Construct and explain Cassegrain's telescope, find its magnifying power, and define the boundaries of the field of view: shew also the advantages of this telescope over that of Gregory.

16. When contiguous parallel rays fall upon a spherical drop of water and emerge parallel after any number of reflections, find the angle between the incident and emergent rays. Explain the forma-

tion of the rainbow, and the cause of the inverted order of the colours in the primary and secondary bows, and determine the breadth of them.

17. Find the form of the surface of a medium which will refract rays diverging from a point without it accurately to a point within itself.

18. The caustic formed by rays after refraction at a plane refracting surface is the evolute to the ellipse or hyperbola, according as the medium is denser or rarer.

NEWTON.

TRINITY COLLEGE, 1820.

1. EXPLAIN by short examples, the method of exhaustions, of indivisibles, and of prime and ultimate ratios.

2. Prove that if a radius vector be drawn bisecting any arc, it must ultimately bisect the chord.

3. If a straight line EDA make with the curve CBA a given angle at the point A, and the ordinates CE, BD be drawn; the triangles ACE, ABD are ultimately in the duplicate ratio of the sides.

4. Let AB be the subtense of the arc, AD the tangent, BD the subtense of the angle of contact perpendicular to the tangent, as in the 11th Lemma: then let a series of curves be drawn in which $DB \propto AD^4, AD^5, AD^6$, &c., the angle of contact in each succeeding case will be infinitely less than in the preceding.

5. If the areas described by the radius vector are not proportional to the times, the revolving body is not acted on solely by a force towards a fixed centre.

6. If a body be acted on by a given force and revolve in a circle, the arc described in any given time is a mean proportional between the diameter of the circle and the space through which a body would descend in the same time from rest if acted on by the same force.

7. The velocity at any point of a curve described round a centre of force = the velocity which a body, acted on by the given force at that point, would acquire by descending through $\frac{1}{4}$ part of the chord of curvature.

8. Given the force of gravity = 32 feet, and the radius of the earth = 4000 miles; deduce a numerical comparison between the force of gravity and the centrifugal force at the equator.

9. If a heavy body be whirled round in a vertical plane, and the centrifugal force at the top just keep the string extended; what will be the tension of the string at the lowest point of rotation?

10. In any orbit, let $x =$ dist. $p =$ perpendicular on the tangent: centripetal force $\propto \frac{dp}{p^3 dx}$. Apply this expression to determine the law of the force in an ellipse round the centre, and in a circle with the centre of force in the circumference.

11. Deduce expressions for the chord of curvature passing through the focus, and the diameter of the curvature at any point of an ellipse.

12. All parallelograms described about any conjugate diameters of a given ellipse or hyperbola are of equal area.

13. Compare the centripetal and centrifugal forces at any point of an orbit; prove that in an ellipse round the centre, there are four points where these forces are equal.

14. Prove [*Newton*, Prop. XI.] that

$$Gv \times vP : Qv^2 :: CP^2 : CD^2.$$

15. The perpendicular from the focus of a parabola upon the tangent is a mean proportional between the focal distances of the point of contact and the vertex.

16. Prove that the

$$\text{force tending to the focus of a parabola} \propto \frac{1}{D^2}.$$

17. The velocity of a body revolving in a parabola round the focus = the velocity of a body revolving in a circle at half the distance.

18. If two bodies revolve in an ellipse in the same periodic time; one about the focus, and the other about the centre; compare the forces towards these centres at the extremities of the major axis, and find the distance from the centres at which the forces are equal.

19. If the force $\propto \frac{1}{D^2}$ and a body be projected in any direction, except directly to or from the centre of force; prove that it will describe a conic section, and point out the relation between the velocity of projection and the particular curve described.

TRINITY COLLEGE, 1820.

1. (1). THE centripetal force (F) in any curve $= Q.\frac{dp}{p^3dx}$, (p) being the perpendicular from the centre of force on the tangent, at distance (x). Determine Q.

(2). Find the value of (F) in the ellipse—the force tending to the centre.

2. If a body be acted on by two forces tending to two fixed centres, it will describe, about the straight line joining those centres, equal solids in equal times.

3. A body describes a parabola about a centre of force situated in the focus:

(1). Find its position at any assigned time.

(2). Given two distances from the focus, and the difference of anomalies. Find the true anomaly.

4. The time of a body's descent, in a right line, towards a given centre of force varies as $\frac{1}{(\text{dist.})^2}$ from that centre. Required the law of the variation of the force.

5. A body at P is urged by an uniformly-accelerating force in the direction PS, and at the same time is impelled in the opposite direction by a force varying as $\frac{1}{(\text{dist.})}$ from S. Find its velocity at any point N.

6. In the logarithmic spiral find an expression for the time of a body's descent from a given point to the centre, and prove that the times of *successive* revolutions are in geometrical progression.

7. A body acted on by a force varying as $\frac{1}{(\text{dist.})^n}$ from the centre, is projected from a given point, in a given direction, and with a given velocity.

(1). Find the equation to the trajectory described.

(2). Determine in what cases the body will fall into the centre, or go off to infinity.

8. The force varying as $\frac{1}{(\text{dist.})^5}$, shew under what restrictions of the velocity of projection, the body's approach towards the centre, and its recess towards infinity, will be limited by *asymtotic circles*.

9. The difference of the forces by which a body may be made to move in the quiescent and in the moveable orbit varies as $\frac{1}{(\text{dist.})^3}$ from the centre.

10. (1). Deduce the equation to the orbit in fixed space.

(2). Shew that when any one of Cotes's three last spirals is made the moveable orbit, the orbit in fixed space will be one of the same species.

11. Why are the principles of the 9th Section inapplicable to the *complete* explanation of the planetary motions?

12. Make a body oscillate in a given hypocycloid.

13. Given the position of a body on a rigid logarithmic spiral which it is made to describe by a force varying as $\frac{1}{(\text{dist.})^2}$ from the pole, find

(1). The point where the body will leave the spiral.

(2). The time of arriving at that point.

(3). The elements of the orbit which it will then describe.

14. Demonstrate the 66th Proposition.

15. Find expressions for the disturbing forces on P when at a given distance from quadratures.

16. If ST and the absolute force of S be changed, the periodic linear errors of P vary as $\frac{1}{(\text{Period of } T)^2}$. [Prop. 66. Cor. 14.]

17. Prove that the mean disturbing force on the moon in a whole revolution $= -\frac{1}{2}$ the mean addititious force.

18. When the force varies as the (dist.), shew that there will be no disturbance.

(1). Had this law pervaded the universe, what would have been the consequence.

(2). On what circumstances in the *variations* of the elements of the orbits does the stability of the planetary system depend?

19. S is the centre, SA the radius of a sphere, each of whose particles has an attractive force varying as $\frac{1}{(\text{dist.})^n}$. Having assumed in SA produced any point P, and having taken $SP : SA :: SA : SI$, find the ratio of the attractions which the whole sphere exerts on equal corpuscles placed at P and I.

20. The attractions of ellipsoids upon particles placed on the surface urging them in directions perpendicular to any principal section are proportional to the distances of the particles from that section.

21. Prove that a shell of homogeneous matter contained between two concentric spherical surfaces, will attract a particle placed without it in the same manner as if all the matter were collected in the centre, in the following cases:

(1). When the law of attraction is that which obtains in nature.

(2). When it varies as the distance.

(3). When it $= Ax + \frac{B}{x^2}$, (x) being the distance.

TRINITY COLLEGE, 1823.

1. Explain the mode of reasoning by which Newton determines the ratios of quantities which vanish together; and prove that the ultimate ratio of the arc, chord, and tangent to each other is one of equality.

2. A body revolves in a semi-ellipse, and is urged by a force acting in lines perpendicular to the axis; find the law of force [Schol. to Prop. 8.]

3. Let $P =$ force on a body at distance r; $u = \frac{1}{r}$; $v =$ angle comprised between r, and a fixed line; $t =$ time of motion; $h =$ twice the area traversed by r in 1″, the following equations will obtain:

$$(1).\ \frac{d^2u}{dv^2} + u - \frac{P}{h^2u^2} = 0.$$

$$(2).\ (\text{velocity})^2 = h^2\left(\frac{du^2}{dv^2} + u^2\right).$$

$$(3).\ dt = \frac{dv}{hu^2}.$$

4. The angle between the distance and perpendicular increases or diminishes according as the velocity in the curve is greater or less than the velocity in a circle at the same distance.

5. Let a body which is acted on by a given force varying $\frac{1}{D^2}$ be projected from a given point in a direction which makes with the initial distance D an angle $= \delta$, and let velocity of projection $= n$ times velocity from infinity,

$$\text{the axis major of orbit described} = \frac{D}{1 - n^2},$$

$$\text{the axis minor} \cdots\cdots\cdots\cdots = \frac{2nD.\sin.\delta}{\sqrt{(1-n^2)}}.$$

6. A body falls down the vertical axis of a paraboloid whose vertex is downwards; in what point of its descent will it have acquired a velocity sufficient to cause it to describe a circle along the surface of the paraboloid, if whirled round as a conical pendulum.

7. The perihelion distance of a Comet's parabolic orbit is a, and the radius of the Earth's orbit, which is circular, is r. The time during which the Comet is within the Earth's orbit is

$$2a^{\frac{3}{2}}\sqrt{\frac{2}{u}}.\left\{\frac{\sqrt{(r-a)}}{\sqrt{a}} + \frac{(r-a)^{\frac{3}{2}}}{3a^{\frac{3}{2}}}\right\},$$

where u = intensity of force at distance unity.

8. If a body be projected from G towards a centre of force S, which varies as $\frac{1}{D^2}$, and velocity of projection $= n$ times velocity in a circle at same distance, n^2 being less than 2; the time of describing GC

$$= \frac{D^{\frac{3}{2}}}{\sqrt{\{(2-n^2)^3.v\}}}\{2\theta - 2\theta' + \sin.2\theta - \sin.2\theta'\},$$

where $D = SG$, $\angle ASD = \theta$, $\angle ASG = \theta'$. [*Newton*, Prop. 37.]

9. A body may be made to describe a lemniscata round a centre of force placed in the nodus, and whose intensity at distance r $= \frac{3h^2a^5}{\gamma^7}$, where $a =$ greatest value of r.

10. A body acted on by a repulsive force which varies $\frac{1}{D^3}$, is projected from an apse with a velocity which is less than that acquired in falling from an infinite distance; construct the orbit. [Prop. 41, Cor. 3.]

11. A body is projected in such a manner as to describe a reciprocal spiral whose equation is $\theta = \frac{a}{r}$: the time of performing the nth revolution $= \frac{a^2}{2(n-1)\pi\sqrt{u}}$.

12. Determine the point of ultimate intersection of mn and pC [Prop. 44].

13. Let the force urging a body consist of two parts, one of which varies inversely as the square, and the other inversely as the cube of the distance; determine the orbit.

14. Explain why the theory of the ninth section cannot be applied to determine the motion of the apsides of the lunar orbit.

15. If the force vary as the distance, determine the nature and equation of the orbit, so that the times of descent from different points in it to the centre, may always be the same.

16. A body acted on by gravity is projected along the interior surface of a cylinder whose axis is vertical; define its motion.

17. Find the relative motion of several bodies which mutually attract each other with a force which varies as the distance.

18. The lunar months are longer in winter than in summer: explain fully the cause of this phenomenon.

19. The mean motion of the nodes is to the mean motion of the apsides in a given ratio, and each varies as $\frac{p}{P^2}$, where $p = P$'s period round T, and $P = T$'s round S.

20. A particle of matter is situated in the axis of a paraboloid consisting of equal particles, each of which attracts with a force

proportional to the distance; find the magnitude and position of a sphere consisting of similar particles, which shall exert equal attraction on it.

21. A particle of matter rests any where on the surface of a solid of revolution, which revolves round its axis in a given time, and is attracted towards the centre with a force which is as some determinate function of the distance from it. Find the form of the solid and apply the result to the case when the force varies directly as the distance.

22. A body acted on by the uniform force of gravity moves in a resisting medium; the resistance to its motion in any point of which the co-ordinates are x, y, is $\frac{-d^3y\,d\dot{s}}{(d^2y)^2}\cdot\frac{y}{2}$, (gravity $= g$), and shew that this result coincides with that of Newton, Book II. Prop. 10.

TRINITY COLLEGE, 1824.

1. STATE the method of reasoning by which Newton determines the ratio of quantities which vanish together. What objection has been raised against the principle, and how does Newton answer it? Shew that quantities which vanish together do not necessarily vanish in a ratio of equality.

2. Define similar curves, when referred to a point, and when referred to an axis: in each case shew that the corresponding sides, curvilinear, as well as rectilinear, are proportionals; and that the areas are in the duplicate ratio of the sides.

3. The ultimate ratio of the arc, chord, and tangent to each other is one of equality.

4. Every body that moves in any curve situated in a plane, and by a line drawn to a point, either at rest, or moving forward with uniform rectilinear motion, describes about it areas proportional to the time, is urged by a centripetal force tending to that point.

5. If h be twice the area traversed by the radius vector in 1″; prove the following expressions:

(1). $$F=\frac{2h^2 . QR}{QT^2 . SP^2}.$$

(2). $$F=\frac{2h^2 . QR}{QP^2 . SY^2}.$$

(3). $$F=\frac{h^2 . dp}{p^3dx}.$$

6. Let a body revolve in the circumference of a circle round a centre of force which is not the centre of circle. Find the law of force.

7. In the preceding proposition, let radius of circle $= r$; distance of centre of force from centre of circle $= a$; and let the distance of the body from centre of force at commencement of motion equal radius of circle. The time elapsed before the body arrives at its least distance from the centre of force

$$=\frac{2r^{\frac{3}{2}}}{(2r^2-a^2)^{\frac{3}{2}}}\cdot\sqrt{\frac{2}{\mu}}\cdot\left\{r^2\theta-a\sqrt{\left(r^2-\frac{a^2}{4}\right)}\right\},$$

where θ is an arc whose sine is $\sqrt{\left(1-\frac{a^2}{4r^2}\right)}$, and $\mu =$ intensity of force at initial distance r.

8. The periodic times in all ellipses round the same centre are equal.

9. The periodic time in an ellipse round the focus

$$=2\pi a^{\frac{3}{2}}\sqrt{\frac{1}{\mu}},$$

where $\mu =$ intensity of force at distance unity.

10. A body describes an equiangular spiral round a centre of force in its centre. Find the law of force.

11. A body describing a circle uniformly, is suddenly impelled in a direction making any acute angle with the radius; and velocity before impact : velocity after :: $\sqrt{2} : \sqrt{3}$. Find the change produced in the periodic time.

12. A body, acted on by gravity and suspended by a string, has a circular motion communicated to it. Shew that the time of revolution is independent of the length of the string, and the same for all cones of the same altitude.

TRINITY COLLEGE, 1826.

1. If SP be the focal distance in a parabola, and PV the abscissa cut off by the semi-ordinate QV; $QV^2 = 4SP \, . \, PV$.

2. All parallelograms formed by tangents drawn to an ellipse at the extremities of any semi-conjugate diameters are equal to each other.

3. The diameters which bisect the lines joining the extremities of the axes of an ellipse are equal and conjugate.

4. Find the polar equations of the parabola, and ellipse; and also of the latter when referred to the centre.

5. The semi-diameter belonging to any point in an ellipse of small eccentricity, is nearly equal to $a\left(1 - \frac{a-b}{a} \, . \, \sin.^2\lambda\right)$, where λ is the angle at which a normal to that point cuts the major axis.

6. The ultimate ratio of the arc, chord, and tangent to each other is one of equality.

7. The evanescent subtense of the angle of contact is ultimately in the duplicate ratio of the subtense of the conterminous arc.

8. State the nature of the angles of contact, to which the reasoning in the first section is restricted; and shew whether there be any such limitation introduced in proving the two preceding propositions.

9. If F, f, be the forces on bodies describing similar parts of similar figures round centres similarly situated in them, and D, d be the central distances, V, v the velocities, P, p the times of describing similar parts of the curves, then will

$$F : f :: \frac{V^2}{D} : \frac{v^2}{d}$$

$$:: \frac{D}{P^2} : \frac{d}{p^2}.$$

10. Shew from the differential equations of motion, that a revolving body attracted by a force tending to a fixed point, will describe round it areas which are in the same plane, and proportional to the time.

11. A body moves in the circumference of a circle, and is attracted towards a fixed point situated without it; find the law of force, and shew through what portion of the circumference the force must be repulsive.

12. A body revolves in the circumference of a circle round a centre of force situated at a distance (a) from the centre of the circle. The periodic time

$$= \frac{2\pi r^{\frac{3}{2}}}{(r-a)\sqrt{\mu}},$$

where μ = intensity of force at least distance.

13. The periodic time in an ellipse round the centre

$$= \frac{2\pi}{\sqrt{\mu}}, \text{ where } \mu = \text{force at dist.} = 1.$$

14. A body acted on by a force which varies $\frac{1}{(\text{dist.})^2}$, is projected from a given point, in a given direction, with a given velocity whose square is less than twice the square of the velocity in a circle at the same distance. Find the orbit described, and its position.

15. If T be the time of a comet's passage after quitting perihelion distance (which $= a$) through an angle $= \theta$, shew that

$$T = a^{\frac{3}{2}} \cdot \sqrt{\frac{2}{\mu}} \cdot \left(\tan.\frac{\theta}{2} + \tfrac{1}{3}.\tan.^3\frac{\theta}{2}\right).$$

16. Sensible gravity at the equator being 288 times the centrifugal force, shew that if the movement of the Earth round its axis were 17 times what it is actually, bodies at the equator would cease to gravitate to the Earth.

TRINITY COLLEGE, 1827.

1. Explain and illustrate by examples the terms Prime Ratio, and Ultimate Ratio.

2. Define similar curves when referred to an axis; and shew that all cycloids are similar curves.

3. At a point A, a straight line AD meets a curve AB. Two parallel lines BD, bd are drawn cutting the curve in B, and b, and

the straight line in D and d. Find the ultimate ratio of the areas ABD, Abd, when B and b move up towards A, (1), when AD is not a tangent to the curve at A, (2) when it is.

4. The path of a body moving about a fixed centre of force is in one plane, and the areas contained by lines drawn from the body to this centre are proportional to the times.

5. A body moves in a circle acted upon by a force tending to a point without the circle. Find the variation of the force, and thence deduce the variation of the force, when it acts in parallel lines.

6. The periodic times in all ellipses described about the same centre are equal.

7. Find the law of force tending to the focus of the hyperbola.

8. Admitting the periods of the different planets to be in a sesquiplicate ratio of the principal axes of their orbits, shew that they are attracted towards the Sun by forces reciprocally proportional to the squares of their several distances from it.

9. In any conic section, the centripetal force is to the centrifugal as the distance of the body from the focus to half the latus rectum.

10. A body, whirled round by a string, just describes a vertical circle, whose radius is the string. What weight must the string be at least able to support, in order to retain the body in the circle?

TRINITY COLLEGE, MAY 1828.

1. EXPLAIN the principles of Newton's method of Limits; and in a right-angled triangle, whose altitude is constant and base variable, find the ultimate ratio of the increment of the base, to the corresponding increment of the hypotenuse.

2. What is meant by the *curvature* of a curve? Show that the curvature at different points of the same circle is the same: and explain why circles are taken to measure the curvature of all curves.

3. Prove Newton's Ninth Lemma.

4. A body moves in a curve, the force tending towards a fixed point; given the velocity and direction of its motion at any point of the curve, to find an equation between the area described, in any time after its leaving that point, and the time of describing it.

5. If F and f be the forces, D and d the central distances at similar points of similar curves, P and p the times of describing similar portions round centres of force similarly situated; shew that

$$F : f :: \frac{V^2}{D} : \frac{v^2}{d} :: \frac{D}{P^2} : \frac{d}{p^2}.$$

6. Investigate the following formulæ :

$$F = \frac{8a^2 . QR}{QT^2 . SP^2}, \quad V = \sqrt{\frac{F . PV}{2}},$$

a being the area described in 1″, and F representing the force on the scale on which that of gravity is represented by $2m$.

7. Two circles, which intersect, are described by two bodies attracted towards a third body, placed in one of the points of intersection: prove, that the periodic times in these circles will be as the cubes of their radii.

8. A body moves in a logarithmic curve, and the force acts in parallel lines, in a direction perpendicular to the abscissæ. Find the law of force.

9. A body revolves in an ellipse, attracted by a force tending to centre. Find the variation of the force: and shew that in different ellipses, periodic time $\propto \frac{1}{\sqrt{(\text{abs. force})}}$. Explain distinctly, what is meant by *absolute* force.

19. $F \propto \frac{1}{D^2}$ and a body, is projected from a given point in a direction which makes an angle (δ) with the initial distance A: then if the velocity of projection $=$ n times that from infinity,

$$\text{the axis-major of orbit described} = \frac{A}{1 - n^2},$$

$$\text{the axis-minor} \ldots\ldots\ldots\ldots\ldots\ldots = \frac{2n A . \sin.\delta}{\sqrt{(1 - n^2)}}.$$

11. Explain the nature of centifrugal force; and prove that in all curves it $= \frac{4a^2}{SP^3}$, (a being the area described in 1′).

12. A body moving in a parabola, force in focus, arrives at the extremity of the latus rectum ($4a$). Show that if (f) represent the force at the vertex, (on the scale on which gravity is represented by $2m$), and a velocity $= \sqrt{\frac{fa}{2}}$ be communicated to the body in the direction of the latus rectum, it will describe a circle round the focus.

Conic Sections.

1. GIVEN the curve and any diameter of an ellipse, and a point in the curve, which is not a vertex of the diameter, to draw from this point an ordinate to the diameter.

2. If any two ordinates QQ', qq', terminated both ways by the curve of a parabola, intersect each other in M; and P, p be their respective parameters, prove that $QM \cdot MQ' : qM \cdot Mq' :: P : p$.

3. Any section of a paraboloid, not perpendicular to the base, is an ellipse.

4. If PQ be an arc of any conic section, QR parallel and QT perpendicular to the radius-vector, the limit of $\frac{QT^2}{QR}$ = the latus rectum.

5. With vertex C, and axis CX, (the centre and asymptote of an equilateral hyperbola) and latus rectum $= CK$ (any distance measured along the asymptote), a parabola is described, cutting the hyperbola in M: show that if KL, be drawn to the hyperbola, and MN to the asymptote, both parallel to the other asymptote, then will MN, CN be two mean proportionals between CK and KL.

TRINITY COLLEGE, MAY 1828.

1. EXPLAIN Newton's method of prime and ultimate ratios, and shew that Leibnitz's positions with regard to curve lines, &c. agree fully with it.

2. What were the circumstances of the motion of the planets discovered by Kepler which served as tests to Newton in his selection of a law for the universal gravitation? How does it appear that the mutual attraction of particles of matter could not be directly as their distance?

3. When a body moves in a curve by the action of a central force, the polar equation to the orbit is

$$u + \frac{d^2u}{dv^2} = \frac{P}{h^2u^2};$$

show that at an apsidal point $u + \frac{d^2u}{dv^2}$ is the reciprocal of the radius of curvature.

4. Explain Newton's method of deducing the circumstances of the rectilinear motion of a body towards a centre, (the force varying inversely as the square of the distance) from those of curvilinear motion. How does he get over the difficulty that on this hypothesis the body should never pass through the centre?

5. Investigate the general Proposition in the 7th Section (Prop. 39), and shew what forms the curves assume when the force varies

(1). as the distance.
(2). ...... square of the distance.
(3.) inversely as the distance.

How is this construction made applicable to the case in which a body is projected towards the centre with a velocity greater than that which it could acquire by falling from an infinite distance?

6. If a body moving in any curve be urged by a centripetal force, the velocity which it has at any place depends solely on its original velocity and its distance from the centre [Prop. 40]. The same proposition is applicable to constrained motion on a curved surface.

7. Investigate in Newton's or Cotes's manner the nature of the orbit described by a body projected with a given velocity, and urged by a force tending to a given centre and varying according to a given law, and apply the result to the case of a force varying inversely as the square of the distance.

8. Explain Newton's method of accounting for the motion of the apsides of an orbit, and prove that the disturbing centripetal force which causes it varies inversely as the cube of the distance from the centre.

9. In Example 3, to Prop. 45, where the force is supposed to vary as $\frac{bA^m + cA^n}{A^3}$, when $T - X$ is substituted for A, it is found necessary to suppose $T = 1$, in order to get out the result. What is the reason of this?

10. **When a body urged by a centripetal force is constrained to move in a plane not containing the centre of force, the equable description of areas obtains with respect to the projection of the centre on the plane. [Prop. 46.]**

11. If two bodies mutually attracting each other revolve about their common centre of gravity; either of them might be made to describe round the other kept immoveable a figure identical with that which it does actually describe relatively to that other.

12. If three bodies be in motion urged by their mutual attractions, there is one case in which their motions take place without any disturbance, and another in which the disturbance is very small.

13. If the motion of a body *P*, revolving about another *T*, be disturbed by the action of a third distant body, (the attractions being inversely as the squares of the distances,) prove that the velocity of *P* will be greatest, the curvature of its orbit least, and its distance from *T* least, in conjunction and in opposition. What is here supposed to be the undisturbed form of *P*'s orbit?

14. In the same case show that the following effects must ensue:

(1). A variation in the periodic time of *P*, if the distance of *S* be variable.

(2). A progression of the apsides of *P*'s orbit, if eccentric.

(3). A variation in this progression.

(4). A variation in the eccentricity of *P*'s orbit.

(5). A variable regression of the nodes of *P*'s orbit, if inclined to the plane containing *T* and *S*.

15. State and explain in order the "Lunar Inequalities" observed by astronomers, before the time of Newton, and show their agreement with the effects mentioned in the last article.

16. What other terrestrial and celestial phænomena does Newton account for in the Corollaries to Prop. 66?

17. How does it appear that Saturn's ring cannot be a connected and rigid body?

18. The common centre of gravity of the Earth and Moon describes about the Sun, very nearly, an ellipse in one plane, and the area passed over by its radius vector is very nearly proportional to the time [Airy's *Tracts*].

19. Prove that if all particles of matter attract each other with forces inversely as the squares of their distances, a corpuscle placed within a spherical shell is equally attracted in all directions Prop. 70.

20. If a spherical shell of indefinitely small thickness composed of particles according to the law of the inverse square of the distance, its attraction on any point without it, is the same as if the whole mass were collected at the centre [Whewell's *Dynamics.*].

TRINITY COLLEGE, June 1829.

1. When are two quantities said to be *ultimately* equal?

Shew that the sine and tangent of a circular arc are ultimately equal, and that the versed-sine ultimately vanishes with respect to the sine.

2. Enunciate and prove Newton, Lemma 4; and apply it, or Lemma 2, to find the area of a portion of a parabola cut off by an abscissa and ordinate to the axis.

3. The spaces described by a body when acted upon by a finite force are ultimately as the squares of the times. Prove this Lemma, shewing clearly that Lemma 9, can be properly applied to this case.

4. Define the circle of curvature, and shew that the chord of curvature to a point P is the ultimate value of $\frac{PQ^2}{QR}$, the chord being parallel to the subtense QR of the angle of contact. Apply this to determine the chord of curvature through the centre and focus of an ellipse.

5. If a body describe equal areas in equal times in the same plane about a fixed point, the body is urged by a force tending to that point.

6. Prove that the centripetal forces in different curves vary as the ultimate values of $\frac{QR}{T^2}$, T being the time of describing an arc PQ; and shew that the force is equal to the ultimate value of $\frac{2h^2\, QR}{SP^2 \times QT^2}$,

and also to $\frac{2V^2}{PV}$, where h = twice the area described in 1″, and V = velocity at P, and PV the chord of curvature through the centre of force.

7. Apply one or other of the preceding expressions for the force, to prove that if F, f, be the forces, R, r, the radii vectores, V, v, the velocities at similar points of similar curves, P, p, the times of describing similar arcs

$$F : f :: \frac{V^2}{R} : \frac{v^2}{r} :: \frac{R}{P^2} : \frac{r}{p^2}.$$

8. A body moves in the circumference of a circle, required the law of force tending to a point without the circle. Through what portion of the circumference must the force be repulsive?

9. If a body revolve in an ellipse, required the law of force tending to the focus. How does Newton infer the converse of this and the two following propositions?

10. Deduce expressions for the periodic time in an ellipse, and for the velocity at any point, force in the focus.

11. A body describing a parabola, force in the focus, arrives at the extremity of the latus rectum. What alteration must be made in the absolute force that the body may afterwards describe an ellipse of which the axis-major shall be equal to the latus rectum of the parabola? Determine also the position and magnitude of the minor axis.

12. If a body describes a curve round a centre of force, shew that $\frac{v^2}{\rho}$ is the resolved part of the centripetal force in the direction of the normal at a point where (ρ) is the radius of curvature, and (v) the velocity of the body. $\frac{v^2}{\rho}$ is sometimes called the centrifugal force. What is the meaning of the term in that case?

13. Apply the differential equations of motion to prove that a revolving body acted upon by a force tending to a fixed point will describe round it areas proportional to the times.

14. If P be a point in a parabola, and QQ' be drawn parallel to the tangent at P and PV parallel to the axis, QQ' is bisected in V.

15. If an ordinate PQ and a tangent PR be drawn from the same point P in a parabola, any diameter DEF terminated by the ordinate and tangent is divided by the curve in the same proportion in which itself divides the ordinate.

16. Shew that in an ellipse $SY^2 = BC^2 \cdot \frac{SP}{HP}$.

17. If a right cone be cut by a plane through both slant sides, the section is an ellipse. Prove this, and having given the points where the cutting plane meets the slant sides, determine the axes of the ellipse.

18. If a tangent be drawn to any point P in an hyperbola, meeting the asymptotes in L and K, and C be the centre of the hyperbola the triangle CLK is invariable.

TRINITY COLLEGE, JUNE 1829.

1. SHEW that the spaces described by a body when acted upon by any finite force are ultimately as the squares of the times. What corollaries does Newton deduce from this Lemma respecting the *errors* of bodies?

2. A body describes an ellipse, required the law of force tending to the focus. [Newton's method.]

3. Find the place of a body describing a given parabola, at an assigned time from its leaving the vertex; also conversely, find the time of the body's arriving at an assigned point. [Newton's method.]

4. S and S' are two centres of force varying directly as the distance, through S a plane is drawn meeting an angle (θ) with SS'; shew that if $SS' = a$, and m, m' be the measures of the forces at distance unity, a body placed at S will oscillate on the plane through a space $\frac{2m' a \cos.\theta}{m + m'}$, in a time $\frac{\pi}{\sqrt{(m + m')}}$.

5. The area of the figure DES described by the indefinite line SD is equal to the area which a body would describe in the same time when revolving uniformly in a circle round S as centre, the radius of the circle being = ½ the latus rectum of the figure DES. [Prop. 35.]

6. Suppose the Moon to be retained in a circular orbit round the Earth's centre, by a force in that centre varying as $\frac{1}{(\text{dist.})^2}$; shew that a heavy body at the Earth's surface acted upon by the same force (diminished according to the above law) would in one second fall through a space $= \frac{2\pi^2 n^3 r}{p^2}$, where $r =$ radius of the Earth (in feet), $nr =$ radius of the Moon's orbit, $p =$ period of the Moon (in seconds), $\pi =$ circumference of a circle where diameter $= 1$. Deduce the numerical value of $\frac{2\pi^2 n^3 r}{p^2}$ in feet, supposing $r = 4000$ miles, $n = 60$, $p = 27$ days, $\pi^2 = 10$.

7. A body acted on by a force varying as $\frac{1}{(\text{dist.})^3}$ is projected from an apse with a velocity acquired down a finite distance; determine the orbit described by the principles of the eighth section, and prove Newton's construction for the orbit.

8. Also when the force varies as $\frac{1}{(\text{dist.})^n}$ determine the differential equation to the curve described, the velocity of projection being (q) times that acquired from infinity; and also the conditions requisite that the orbit may have an asymptotic circle.

9. Deduce the equation to the orbit VCp [Section 9], that of VCP being known, and apply it to shew that the difference of the forces on P and p varies as $\frac{1}{(\text{dist.})^3}$. If VCP be an ellipse of small eccentricity, and C the centre, compare the radii of curvature in the points of the orbit VCp corresponding to the extremities of the axes of the ellipse.

10. Explain fully Newton's method of determining the angle between the apsides in orbits nearly circular, and exemplify in the case where the force varies as $\frac{1}{a^2 + r^2}$, r being the radius vector. Why cannot the principles of the ninth Section be applied to determine the amount of the progression of the Lunar apogee?

11. If a body oscillate in an hypocycloid, acted upon by a force tending to the centre of the globe and varying as the distance, the oscillations will be isochronous.

12. A body acted upon by gravity moves on an inverted cone with its axis vertical, find the equation to the projection of the orbit on a horizontal plane, and the law of the force tending to the vertex by which that projection might be described.

13. If P and S attracting each other with forces varying as $\frac{1}{(\text{dist.})^2}$ revolve round their common centre of gravity, shew that the apparent orbit of one round the other is an ellipse; and compare the major axis of the ellipse which P describes relatively to S in motion with that which P would describe in the same time about S at rest.

14. If the motion of a body P revolving about another T be disturbed by the action of a third distant body, explain fully the general effects produced,

(1). On the periodic time of P, the distance ST being variable. [Prop. 66, Cor. 6.]

(2). On the eccentricity of P's orbit. [Cor. 9.]

(3). On the motion of the nodes of P's orbit, if inclined to the plane containing T and S. [Cor. 11.]

15. Find the resolved part of the Sun's disturbing force on the Moon in the direction of the projection of the radius vector on the ecliptic [Airy's Tracts]; and determine its approximate value supposing the Moon to move in the ecliptic, and to be at an angular distance (θ) from syzygy.

16. Shew that if P be the period of T round S, and p that of P round T, the mean addititious force of S on P : mean force of T on P (that is, the force by which P would in time (p) revolve round T at distance PT) :: $p^2 : P^2$; and that if V be the mean velocity of P in a circular orbit about T, and (v) its velocity at an angle θ from quadratures, a near approximation to its velocity at an angle (θ) from quadratures will be

$$V - \tfrac{3}{4}\frac{p^2}{P^2} V \cos.2\theta.$$

17. Having given the attraction of a sphere on a particle without it, find its attraction on a particle situated within the sphere; the attraction of the particles composing the sphere varying as $\frac{1}{(\text{dist.})^n}$.

18. Two straight lines intersect at right angles, determine the attraction of one of them on a given portion of the other,

$$\text{Attr}^{\text{n}} \propto \frac{1}{(\text{dist.})^2}.$$

19. Find the attraction of an oblate spheroid of finite, and also of small eccentricity on a particle at its equator.

20. If the Earth's equator be surrounded by a canal of water, determine the form which the waters will assume when acted upon by two distant bodies as the Sun and Moon in the plane of the equator, the attraction on any point independently of the disturbing forces of the Sun and Moon varying as its distance from the Earth's centre.

TRINITY COLLEGE, MAY 1830.

1. WHAT laws of motion, first discovered by observation at the Earth's surface, did Newton in his theory of universal gravitation, extend to the motions of the heavenly bodies?

2. The spaces described by a body, urged by any finite force, are at the beginning of the motion in the duplicate ratio of the times.

3. Admitting Kepler's law of the equable description of areas, shew that the force, acting upon a planet, is directed to the Sun.

4. From another of Kepler's laws, the elliptic form of a planet's orbit, infer that the force varies inversely as the square of the distance from the Sun, and that the square of the periodic time varies as the cube of the mean distance.

5. Having given 1 : 390, the ratio of the Moon's distance from the Earth to the Earth's distance from the Sun, and $365\frac{1}{4} : 27\frac{1}{3}$, the ratio of the periodic times of the Earth and Moon, prove that the ratio of the mean force acting on the Earth to that on the Moon, is 2·17 nearly.

6. The time of describing any parabolic arc PQP', of which PMP' is the chord, QM the sagitta, and S the focus, is equal to

the time of describing the chord uniformly with the velocity at the distance $\frac{(SQ + \frac{1}{3}QM)^2}{SQ}$ from the focus. [*Newton.*]

7. Obtain three terms of a series for calculating the true anomaly from the mean.

8. Required the equation of the orbit described, and the angle between the apses, when the force is $mr + \frac{m'}{r^3}$.

9. If a body move on a surface of revolution by the action of a force, the centre of which is situated in the axis, the projection of the radius-vector joining the centre and the body, upon a plane perpendicular to the axis, will describe equal areas in equal times.

10. If two bodies be projected with any velocities in any directions in space, and the consequent motion of their centre of gravity be impressed on them in a direction contrary to that in which it takes place, the remaining motions will be in one plane, parallel, and inversely proportional to the masses; and the bodies, mutually attracting, will describe about each other and their centre of gravity, similar orbits.

11. The orbit which two bodies mutually attracting describe relatively to each other, may be described in the same time by a material particle revolving round an attracting mass at rest, equal to the sum of the bodies.

12. Required the approximate values of the central and tangential disturbing forces, when the orbit of P, supposed circular, is inclined to that of S at a given angle; also the disturbing force acting perpendicularly to the plane of P's orbit.

13. What are the causes of the *variation* and the *annual equation*, and when are these inequalities greatest?

14. State the cause of the inequality called *evection*; and shew that the eccentricity of the Moon's orbit increases as the apsidal line moves from quadratures to syzygies, and diminishes as it moves from syzygies to quadratures.

Of the three inequalities mentioned in the two preceding questions, which most affects the Moon's place, which least?

15. Obtain, according to Newton's method, the angle between the apses of an orbit nearly circular in which the force is $\frac{m}{r^2} - \frac{Mr}{2R^3}$, the mean of that tending from the Moon to the Earth ; and having given 179, the square of the ratio of the Earth's to the Moon's periodic time, perform the numerical calculation of the angle.

16. Similar angular errors in different systems vary as

$$\frac{(\text{period of } P)^2}{(\text{period of } T)^2}.$$

In what manner does the direct solution of the Problem of Three Bodies shew, that this Proposition is much more true for the motion of the nodes than for that of the apses?

17. Taking for granted a result which Laplace has arrived at; viz. that if m be the mass of Jupiter, m' of Saturn, and a, a', be their respective mean distances from the Sun, $\frac{m}{a} + \frac{m'}{a'}$ remains constant notwithstanding their mutual perturbations ; prove that corresponding small variations in the mean angular motions of the two planets, will have different signs, and will be nearly in the ratio of 3 to 7, if $\frac{m}{m'} = \frac{34}{11}$ and $\frac{a}{a'} = \frac{52}{95}$.

18. If D be the interval between the centres of two spheres, attracting according to the law of gravity, M, M', be their masses, and if the density in each be any function of the distance from the centre, the moving force with which they attract each other will be $\frac{M \times M'}{D^2}$. [*Newton*, Sect. xii.]

19. Prove that the form of the oblate spheroid, which a fluid mass revolving with a given angular velocity, will take, is such that the force at the pole is to that at the equator, as the radius of the equator to the radius at the pole.

20. A particle placed any where within the interior surface of a solid shell, which is bounded by similar and concentric ellipsoidal surfaces, and attracts according to the law of the inverse square of the distance, will be at rest.

21. If 1, m, m' be the masses respectively of three bodies S, E, M, which attract each other according to the law of gravity; if x, y, z, r,

be the co-ordinates and radius-vector of M, having their origin at S, which is supposed to be at rest; and if x', y', z', r', be the analogous lines with respect to E; then will the motion of M be given by the solution of the equations

$$\frac{d^2x}{dt^2} + (1 + m')\frac{x}{r^3} + m\frac{dR}{dx} = 0,$$

$$\frac{d^2y}{dt^2} + (1 + m')\frac{y}{r^3} + m\frac{dR}{dy} = 0,$$

$$\frac{d^2z}{dt^2} + (1 + m')\frac{z}{r^3} + m\frac{dR}{dz} = 0;$$

where

$$R = \frac{xx' + yy' + zz'}{r'^3} - \frac{1}{\sqrt{\{(x - x')^2 + (y - y')^2 + (z - z')^2\}}}.$$

TRINITY COLLEGE, MAY 1830.

1. DEFINE the terms *parabola, ellipse, hyperbola, diameter, parameter, latus rectum.*

2. In the parabola, the subnormal is equal to half the latus rectum.

3. The rectangle under the latus rectum of the parabola, and the part of any diameter cut off by an ordinate to the axis, is equal to the rectangle under the segments of the ordinate.

4. The tangent to an ellipse makes equal angles with lines drawn from the point of contact to the foci.

5. The rectangle under the perpendiculars from the foci upon the tangent is equal to the square of the semi-axis minor.

6. The lengths cut off from a normal by the axis major, conjugate diameter, and axis minor, are in continued proportion.

7. If tangents be drawn at the vertices of four conjugate hyperbolas, forming a rectangle, the diagonals of this figure are asymptotes to the four curves.

8. Find, in each of the Conic Sections, the value of the chord of curvature through the focus.

9. Prove, by the definition given in answer to the first question, that the curve, produced by the oblique section of a cone by a plane, is an ellipse.

10. Explain fully Newton's first Lemma.

11. Prove that the areas of similar figures, bounded wholly or partly by curved lines, are to one another in the duplicate ratio of their homologous lines.

12. Demonstrate that in all curves of finite curvature, the evanescent subtense of the angle of contact is ultimately in the duplicate ratio of the conterminous arc. What is meant by *finite curvature?*

13. If a body in motion be affected solely by a force tending constantly to a fixed point, or centre, the sectorial areas which it describes about that point must be proportional to the times of description.

14. Explain and demonstrate Cor. 8, Prop. IV. "Eadem omnia de temporibus, velocitatibus, et viribus, quibus corpora similes figurarum quarumcunque similium, centraque in figuris illis similiter posita habentium, partes describunt, consequuntur ex demonstratione præcedentium ad hosce casus applicatâ."

15. Given the velocities of a planet at three given points of a given orbit: required the centre of force.

16. If α represent the area described in the unit of time, the force at any point P of an orbit is measured by $8\alpha^2 \dfrac{QR}{SP^2 \cdot QT^2}$, or by $\dfrac{2V^2}{X}$, V representing the velocity, and X the chord of curvature through the centre.

17. Determine the law of force by which an equiangular spiral might be described.

18. In different elliptic orbits, having a common focus, the squares of the periodic times are as the cubes of the major axes.

ST. JOHN'S COLLEGE, 1824.

1. DEFINE what is meant by the terms "limit" and "limiting ratio" and from your definition show that the limiting ratio of the chord, arc, and tangent to each other is a ratio of equality, when the arc itself vanishes.

2. AB, AF are two straight lines intersecting each other in A, and S is any other point. Take a point E in the line AF; divide AE into three equal parts AC, CD, DE; join SC, SD, SE cutting AB in (m), (m') and (m''), and find the limiting ratio of mm' to $m'm''$, when AE is indefinitely increased.

3. Prove, (what Newton has assumed), that in an ellipse

$$Pv \cdot vG : Qv^2 :: CP^2 : CD^2,$$

and from it show that if a body revolves in an ellipse (force in focus), the force $\propto \frac{1}{(\text{dist.})^2}$.

4. Show that, if two equal centres of force, varying as the distance, be placed in the two foci of an ellipse, a body projected with a proper velocity may be made to describe this ellipse.

5. Explain what is meant by the expression, "velocity acquired in falling from an infinite distance," and show from thence geometrically, that, when force $\propto \frac{1}{(\text{dist.})^3}$, the velocity in a circle $=$ this velocity.

6. Find the actual velocity of a body revolving in an hyperbola, when force $\propto \frac{1}{(\text{dist})^2}$, and find the limit to which it approximates, as the body recedes from the centre of force.

7. Suppose a comet, whose radius $= r$, to revolve in a parabola, and at its perihelion just to touch the surface of the Sun, and to lose in consequence $\left(\frac{1}{n}\right)^{\text{th}}$ part of its velocity, find the eccentricity of the ellipse it will afterwards describe, and compare the areas described d.t. before and after impact.

8. If a stone be thrown with a given velocity from the Moon, in what direction must it be projected that in its course it may just clear the surface of the Earth, and what is the least velocity of projection sufficient for this purpose? (The attraction of the Moon and the resistance of the Earth's atmosphere being supposed inconsiderable.)

9. A circular flexible ring revolves in its own plane about its centre with a given angular velocity. Find the tension, arising from the centrifugal force.

10. A body is describing a given ellipse about a sphere of matter, whose radius is greater than the least focal distance of the ellipse, and through which it is supposed to pass without resistance. Required the alteration in the position of the orbit after passing through the sphere, and the orbit described during its passage through. (The force without the sphere varying inversely as $(\text{distance})^2$, and within directly as distance.)

ST. JOHN'S COLLEGE, DEC. 1825.

1. STATE what is the analytical method of finding the ultimate ratio of evanescent quantities. Apply it to finding the value of $\frac{x^2 + y^2}{x}$ when x and y vanish, x being the abscissa and y the ordinate of a curve which passes through the origin at right angles to the axis, and shew from the result that it is the diameter of curvature at the origin.

2. The area of the cissoid of Diocles between the vertex and an ordinate to the axis : its circumscribing triangle :: 4 : 5 ultimately.

3. A line drawn through the middle point of an arc of continued curvature, and parallel to the chord, is ultimately a tangent.

4. Having given the velocity and direction of a body at two points of its orbit, find the locus in which the centre of force must be situated.

5. A flat ring, the internal radius of which is given, revolves in its own plane above its centre with a given angular velocity; find

the magnitude and law of force tending to its centre round which it would revolve in the same manner if the cohesion of the particles were destroyed.

6. Compare the centrifugal forces at similar points of similar curves, round centres of force similarly situated.

7. If a body describe a semicircle by a force tending to a point so remote that all lines drawn from the circle to it may be considered parallel, shew that the force at any point P varies reciprocally as PM^3. [Section II. Prop. 8.]

8. If the lines drawn to the circle from the centre of force, in the same proposition, be only very nearly parallel, shew that the force at any point P must be increased by a quantity which varies as $\frac{1}{PM^2}$.

9. The area which a body describes round a centre of force in the time $t = \frac{1}{2} . ptv$, p being the perpendicular corresponding to the velocity v.

10. The force in any conic section tending to the centre of force $R \propto \frac{CG^3}{RP^2}$; C being the centre, and CG parallel to the distance RP meeting the tangent at P in G; [Scholium, Section III.] Prove this, and shew that if R be the vertex of an ellipse, the force $\propto \frac{RP}{RN^3}$, PN being perpendicular to the major axis.

11. There are three equal bodies, two of which describe circles round the third fixed, in such a manner that the bodies are always in the same right line: compare the radii of the circles, with the three bodies attracting each other with forces which vary as $\frac{1}{\text{dist.}}$

12. If the central force of a body moving in a parabola round the focus, were to cease acting at the vertex, and continue interrupted till the body had described an angle of 60° above the focus; find the eccentricity of the orbit it would afterwards describe.

St. JOHN'S COLLEGE, 1826.

1. A SEGMENT of a sphere : its inscribed cone :: 3 : 2 ultimately.

2. Shew how to find when the curves whose equations are obtained by altering one constant in a given equation are similar, and find whether the curves formed by giving different values to a in the equation $y^2 - ax + ab = 0$ are similar.

3. If the Earth be struck by a comet of a given mass, which moves in the plane of the ecliptic, and the perihelion distance of which is equal to the radius of the Earth's circular orbit, find the alteration produced in the length of the year.

4. If the least distance of the comet be not quite equal to the radius of the Earth's orbit, find how the length of the day is altered supposing the ecliptic and equator to coincide.

5. Find the equation to the curve in which the centrifugal and centripetal forces are in a given ratio.

6. Compare the forces at similar points of similar curves; the force acting in parallel lines and the velocities in a direction perpendicular to that in which the force acts being the same in the different curves.

7. Compare the periods in two equal circles round the same centre of force which is situated in the centre of one circle, and in the circumference of the other.

8. If a body move from one point to another acted on by any centripetal force, shew that the velocity is independent of the curve in which it moves.

9. The velocity is also independent of the curve if the body be acted on by any number of centripetal forces. Does the proposition hold for all forces not centripetal?

10. Explain Newton's method of finding the angle between the apsides in an orbit nearly circular. Find the angle between the apsides in the curve whose equation is $\frac{1}{r} = C \cos.\gamma v + C' \sin.\gamma v$, r being the radius vector, v the angle, and C, C', γ constants.

11. Find the disturbing force of the Sun perpendicular to the plane of the lunar orbit.

12. If a body move freely on the surface of a solid of revolution acted on by forces tending to all points of the axis, shew that the angular velocity round the axis varies inversely as the square of the distance from the axis.

13. Explain why the Sun acting on the protuberant matter of the Earth's equator causes a precession of the equinoxes.

ST. JOHN'S COLLEGE, DEC. 1826.

1. THE ultimate ratio of evanescent quantities is a ratio of equality, when their difference is an evanescent of an higher order than themselves; explain this phrase, and shew that the rule is contained in the *words* of the first Lemma; the term *data differentia* being understood a quantity of finite magnitude.

2. Shew from the preceding question that the axiom, "If equals be taken from equals, the remainders are equal," does not necessarily hold when applied to the Geometry of the first Section, and give an example.

3. Define similar curves when referred to polar co-ordinates; prove that curves represented by the equation $\theta = \phi \frac{r}{a}$ are similar, and hence determine on what condition different conchoids will be similar.

4. In a curve of finite curvature, the chord, arc, and tangent, are ultimately in a ratio of equality.

5. AB is the chord of an arc ACB of finite curvature, take $AC : CB$ always as $m : n$; join AC, BC, and prove that the $\triangle ABC$: segment $ABC :: 3 : \left(\sqrt{\frac{m}{n}} + \sqrt{\frac{n}{m}}\right)^2$ ultimately.

6. Given three points in an orbit and the three corresponding angular velocities; to find the centre of force.

7. Find the law of force, that the velocities in different circles about the same centre may be the same.

8. A body revolving in a ⊙ sets out from *A*; required to draw the chord *AF*, along which a body descending by the force in the circumference acting *uniformly parallel* to the radius *CA*, may meet the revolving body again in *F*.

9. Prove fully that the force in any curve varies inversely as $SY^2 . PV$; when *SY* is the ⊥ on the tangent, and *PV* the chord of curvature through the centre of force. And apply the expression to find the law of force in the reciprocal spiral.

10. If a body be projected from a given point with the same velocity in different directions, and be acted on by a force tending to a given point, which varies inversely as D^2; find the curve, which is the locus of the centres of all the ellipses described.

11. Prove that when the velocity in any curve equals velocity in the ⊙ *e . d*, the angular velocities of the radius-vector and ⊥ on the tangent are equal, and find when this takes place in a ⊙, force not in the centre.

12. Force to *C* varies as distance. A body is projected from *P* along the line *zPy*, with velocity $= m$. times velocity in ⊙ rad. *CP*. Draw $PK = m . CP$, making the $\angle zPK = CPy$ and join *CK*. Shew that the axis-major of the ellipse described bisects the $\angle PCK$, and complete the construction, so as to determine the magnitude of the axes.

13. A body is revolving in an ellipse; the centre of force is suddenly transferred from the focus *G* to *H*. Shew that if at the same instant, the body be so situated, that the normal divides *SH* in extreme and mean ratio, it will describe a parabola, and find its latus-rectum.

14. If a comet describing a parabolic orbit inclined to the plane of the ecliptic, be seen to pass over the Sun's disc, and $\frac{1}{4}$ of a year after, to strike the planet *Mars*, whose distance from the Sun is $\frac{3}{2}$ that of the Earth; required the distance of the comet from the Earth at the first observation in parts of the radius of the Earth's orbit, supposed circular, and its plane coincident with that of Mars.

ST. JOHN'S COLLEGE, MAY 1827.

1. PROVE that the force in any orbit varies as limit $\frac{QR}{T^2}$, where QR is the subtense of an arc drawn parallel to the force, and T is the time of moving through it.

2. Find the law of force parallel to the base, by which a body may move in a cycloid, and find when the force is least.

3. Required the position of a plane, against which a corpuscle revolving in a circle, at a distance equal twice the Earth's radius must impinge, that after reflection it may just pass round the Earth, elasticity being perfect; and find the time it remains within its former orbit.

4. Determine what Newton's trochoid in the sixth section becomes, when the axis minor of the ellipse vanishes; and reconcile the construction with that given in the seventh section, for the time in a descent towards a centre of force, which varies inversely as the square of the distance.

5. Force $\propto \frac{1}{D^3}$. A body is projected with a velocity which is to that in a circle at the same distance as $\sqrt{2} : 1$, and in a direction making an angle of 30° with the distance; find the equation to the orbit reckoned from the asymptote, draw the asymptote and find the time of descent to the centre.

6. If the fixed orbit, which is a parabola, force in focus, be projected in antecedentia with an angular velocity equal to half that of the body in the fixed orbit; required to trace the curve formed; find its point of contrary flexure, and the point where the forces in the two orbits are as $2 : 1$.

7. Equal absolute forces are placed in A and B which vary as the distance. Required the direction and velocity with which a body must be projected from a given point, so as to describe an ellipse having A and B in the foci.

8. Investigate an approximate expression for the ablatitious force in Prop. 66, and shew that it equals three times the addititious force × sine of the angular distance of P from quadratures.

9. Explain what is meant by the Moon's variation; give Newton's construction for it, and determine when it is a maximum.

10. Investigate the path of a body projected at a given angle in a medium whose resistance varies as the velocity, and acted on by the force of gravity. Find its greatest altitude, and the resistance at that point.

11. The centre of one bounding surface of a meniscus of matter coincides with the extremity of the diameter of the other; required the attraction on a corpuscle placed on that point, the force to each particle varying as $\frac{1}{D^2}$.

12. Shew that, from the effect of centrifugal force, a plane circle cannot revolve permanently about an axis through its centre inclined to its plane; and find the quantity of matter in an uniform rod, whose length equals the diameter, to be fixed to it through its centre at right angles to its plane, that the whole may revolve permanently about any axis through the centre.

St. JOHN'S COLLEGE, May 1828.

1. Find the limiting ratio of the solid formed by the revolution of the segment of a circle about its base, to the solid formed by the circumscribing rectangle, when the segment is diminished without limit.

2. If a number of bodies describing different circles about the same centre of force, which $\propto \frac{1}{D^3}$, set out together from the same radius; required the curve in which they all will be found, when that body, the radius of whose orbit is (a), has completed its revolution.

3. Prove geometrically, that the force acting in parallel lines, which would cause a body to move in any curve, is to the force which, acting in a direction perpendicular to the former, would cause it to describe the same curve, :: rad.^{3} : tan.^{3} of the angle, which the curve makes with the first direction.

4. Give Newton's construction for the time, and velocity, when a body falls towards a centre of force, which varies as the distance: and shew that the velocities acquired in falling through different heights to the centre, are as the height.

5. Force $\propto \frac{1}{D^3}$. A body is projected from a given point with a velocity greater than that which could be acquired by falling from an infinite distance. Distinguish the different orbits which may be described, and the conditions in each. Also find the ratio between the centripetal and centrifugal forces in each case.

6. Investigate the effect of the disturbing force of the Sun upon the node line of the lunar orbit. [Prop. 66. Cor. 11.]

7. The time of a revolution of the Moon's nodes being 19 years, what would be the time of revolution of the nodes of a satellite describing an orbit nearly circular, at ¼th part of the distance of the Moon.

8. A body is constrained to move in a parabolic arc, acted on by a force perpendicular to the axis. Required the law of the force, that it may recede uniformly from the axis, and also the law of the pressure.

9. The first satellite of Jupiter performs its revolution in 1 day, 18 hours, at a distance equal to six times the radius of the planet; Jupiter revolves about his axis in 10 hours. Determine hence the ratio of the axes of the planet, considering it an homogeneous spheroid of small eccentricity.

10. If within a given sphere, whose particles attract each other with forces varying inversely as the square of the distance, there be inscribed a concentric spherical surface; required the attraction of the smaller sphere to a segment of the larger, cut off by a tangent plane to the former. Also required the radius of the smaller sphere, when this attraction is a maximum.

11. If the resistance of a medium vary as the density into the square of the velocity, prove that the orbit described by a body about any centre of force is to be determined by the integration of the equation,

$$\frac{d^2u}{dv^2} + u - \frac{P \cdot \varepsilon^{2\int Q ds}}{h^2 \cdot u^2} = 0, \text{ where } u = \frac{1}{r},$$

v is the angle described, P the centripetal force, and Q the density.

12. Construct for the horary variation in the inclination of the Moon's orbit, and shew that the mean variation during one lunation, $\propto$ sine twice the angular distance of the nodes from quadrature, into sin. inclination.

ST. JOHN'S COLLEGE, MAY 1829.

1. The force tending to a point S, by which a body is retained in its orbit, is equal to $\frac{2h^2}{SP^2} \times$ limit of $\frac{QR}{QT^2}$: where h = twice the area described in a second.

2. Given the radius of the Earth, the force of gravity at its surface, and periodic time of the Moon, required her distance from the Earth.

3. Determine the space due internally to the velocity of a body describing a circle round a centre of force in the circumference.

4. A body moving in an elliptic orbit, the force varying as the distance, leaves an apse: what is the distance of the body from the centre when an eighth part of the periodic time has elapsed?

5. Explain Newton's method of finding the apsidal angle in an orbit nearly circular, and exemplify it when the force $\propto \frac{ar^m + br^n}{r^3}$.

6. A body describes a circle by the action of a force tending to its centre, varying inversely as the seventh power of the distance. If the absolute force be increased threefold, what will be the new orbit described by the body, and the interval before it arrives at the centre?

7. The force of gravity without the surface of the Earth varies inversely as the square of the distance from the centre; and within it, directly as the distance.

8. Given the radii of the orbits of the Earth and Moon, and their periodic times; required the time in which the Moon would perform a revolution round the Sun, if, when she is in conjunction, the Earth were annihilated.

9. A body attracted to a point by a force $= \frac{r}{m^2} + \frac{h^2}{r^3}$ is projected from an apse at the distance $\sqrt{(mh)}$; h being twice the area described in a second. Find the polar equation to the orbit, and shew that the time of describing an angle $\theta = m \,.\, \tan.^{-1} \theta$.

10. A body acted upon by a constant force in parallel lines is projected in an uniform medium, of which the resistance varies as the velocity; required the greatest altitude to which it will ascend.

11. The Earth, another planet, and a comet move in the same plane round the Sun. The perihelion distance of the comet is the difference between the radii of the orbits of the Earth and planet, and the duration of the comet's stay in the Earth's orbit is n times the periodic time of the planet. Compare the distances of the Earth and planet from the Sun.

12. Required the horary motion of the Moon's nodes in a circular orbit.

ST. JOHN'S COLLEGE, JUNE 1829.

1. THE homologous sides of all similar figures, whether curvilinear or rectilinear are proportionals, and the areas are in the duplicate ratios of the sides.

2. If AD touch the curve ACB of continued curvature in the point A, and BD be drawn meeting AD at a finite angle, and the arc ACB be continually diminished, the limiting ratios of the chord, arc, and tangent, to each other, are ratios of equality.

3. The spaces described from rest by a body acted on by any finite force are, in the very beginning of the motion, as the squares of the times.

4. If a body moving in a plane curve describes areas proportional to the times by lines drawn from the body to any point, the body is retained in the curve by a force tending to that point.

5. The space due to the velocity at any point of a curve by the action of that force continued uniform is a quarter of the chord of curvature at that point.

6. Force $\propto \frac{1}{SY^2 . PV}$.

7. Find the law of force tending to the focus by which a body may describe a parabola.

8. If a body be projected from P in the direction ZPR, which makes any angle with the distance SP, and be urged by a force tending to S, varying inversely as the square of the distance, it will describe a conic section, of which S is the focus.

9. The periodic time of a body describing an ellipse round a centre of force in the focus $= \frac{2\pi AC^{\frac{3}{2}}}{\sqrt{m}}$.

10. To find how far a body must fall externally and internally by the action of a variable force, to acquire the velocity of a body moving in an ellipse round a centre of force in the focus.

11. To determine the time in which a body falling from rest from any given point A will descend through any given space AC, by the action of a force at S varying inversely as the square of the distance.

12. If the force varies directly as the distance, the time of descent and the velocity acquired from rest, are respectively proportional to the arc and sine of that arc, whose radius is the whole distance from which the body begins to fall, and versed sine, the space fallen through.

ST. JOHN'S COLLEGE, DEC. 1829.

1. DEFINE a limit; and shew that if, as x increases, $f(x+1) - fx$ converges to a limit, then $\frac{1}{x} fx$ converges to the same limit. What is the limit of $\frac{1}{x} \log. x$, when x is infinite?

2. If a body revolve in an ellipse about the focus, its velocity will be an arithmetic mean between the greatest and least velocities, when $\cos. ASP = -\frac{e}{2}$, e being the eccentricity, and AS the least distance.

3. At what point of an hyperbola, are the angular velocities about the foci in a given ratio? and what is the limit of that ratio? $F \propto \frac{1}{D^2}$.

4. If P be a point in a conic section whose focus is S, and G the point where the normal at P meets the axis, shew that if GK be drawn perpendicular to SP, $PK = \frac{1}{2}$ latus rectum; and hence construct the orbit described by a body, projected from a given point with known velocity and direction, about a centre of force $\propto \frac{1}{D^2}$.

5. Two planets describe circles in the same plane about the Sun whose mass is known; given their distances from the Sun, find the interval between their being at the greatest and least distances from one another.

6. As a body revolves in a parabola about the focus, a tangent is always drawn from it, meeting AB the tangent at the vertex in Q; shew that the velocity of Q along AB varies inversely as the distance of the body from the centre of force.

7. If v be the velocity, and f the centripetal force, at any point of a curve whose distance from the centre of force is r; and v', f', r', similar quantities at the corresponding point in the locus of the intersection of the tangent and perpendicular upon it from the centre,

$$\text{then, } \frac{v^2}{rf} + \frac{r'f'}{v'^2} = 2.$$

8. "Posita cujuscunque generis vi centripeta, et concessis figurarum curvilinearum quadraturis, requiritur corporis recta ascendentis vel descendentis tum velocitas in locis singulis, tum tempus quo corpus ad locum quemvis perveniet." [Prop. 39.]

9. If two bodies whose masses are m, m', be placed on a smooth horizontal plane, and be connected by a string passing through a small ring in that plane; and if m be projected with a velocity (α) in a direction perpendicular to its distance (a) from the ring, then, when its distance is r, shew that

$$\text{the tension of the string} = \frac{mm'}{m + m'} \cdot \frac{(a\alpha)^2}{r^3}.$$

10. In the last question, determine the time which m' takes to reach the ring, and the acquired velocity.

11. If a comet, besides being attracted by the Sun, were disturbed by a constant force acting at every point in the direction of a diameter, it might still describe the same orbit; and the velocity at any point would be that due to the focal distance by the action of both forces continued uniform.

12. If v be the velocity of a body revolving in an ellipse about the centre, v' its velocity when the direction of its motion is at right angles to the former direction,

$$\text{the time of describing the intercepted arc} = \frac{1}{\sqrt{m}} \sin.^{-1} \frac{vv'}{mab},$$

where m is the absolute force, and a, b, the $\frac{1}{2}$ axes.

St. JOHN'S COLLEGE, May 1830.

1. Construct for the place of a body, at the end of a given time, in an ellipse round the focus.

2. Bodies are projected from a given point, with the same velocity in different directions, and acted on by a central force which $\propto$ (distance)$^{-2}$: find the nature of the surface which touches all the conic sections described.

3. Two spheres attracting with forces which $\propto$ (distance)$^{-2}$, are projected at the same instant, with the same velocity and in the same plane, in a direction perpendicular to the line joining their centres: determine their motions.

4. Find the angle between the apsides of the orbit described by a body in a horizontal plane, round a point in the plane to which it is attached by means of an elastic string.

5. Find the polar equation to the Moon's orbit, on the supposition that the Moon's gravitation $\propto$ quantity of light received from the Earth.

6. Find the mean annual motion of the Moon's nodes.

7. Determine the motion of a body descending from rest in the sea, the density of the Earth being supposed uniform, and equal to that of water.

8. Determine the motion of a body projected in a medium, in which the resistance is as the square of the velocity, and acted upon

by gravity. Prove that $v^2 = \frac{1}{2} f . c$ where v is the velocity, f the accelerating force, and c the vertical chord of curvature, at any point in the trajectory.

9. Having given the times of the small oscillations of a pendulum about each of three given parallel axes, in a plane passing through its centre of gravity, shew how the length of a simple seconds pendulum may be found.

10. Determine the time of the very small oscillations of a convex body rolling on a concave surface.

11. Having given $\alpha = \frac{3}{r^3} . B . S . \sin.\theta . \cos.\theta$, investigate the quantity of solar precession for any given time.

12. A slender ring is composed of matter attracting with a force which $\propto$ (distance)$^{-2}$: find the angular velocity with which it must revolve in order that it may experience no tension.

13. Determine the ellipticity of a planet accompanied by a satellite.

CORPUS CHRISTI COLLEGE, June 1829.

1. Define angle of contact, and subtense of angle of contact; and shew that if there be any two curves *AB*, *Ab*, in which the angle of contact *BAD* bears a finite ratio to the angle of contact *bAD*. Then if the curvature of *AB* be finite, the curvature of *Ab* will also be finite, and conversely.

2. The spaces which a body by action of any finite force describes, whether that force be constant and invariable, or continually increased or diminished, are in the very beginning of the motion in the duplicate ratio of the times.

3. In all curves which have a finite curvature at the point of contact, the ultimate ratio of the subtenses of the angle of contact is the same with that of the squares of the coterminous arcs.

4. The centripetal forces of bodies which describe any similar curves round centres of force similarly situated within them are to each other as the squares of the arcs described in equal times directly and the radii inversely.

5. The space through which a body must fall to acquire a velocity equal to the velocity in the curve at any point equals one fourth of the chord of curvature at that point, the accelerating force being the force at this point continued constant. Prove this, and compare the velocity in the curve whose equation is $u^2 = m^2 \cos.2\theta$ with the velocity in a circle at same distance.

6. A body revolves in the circumference of a circle, find the law of force tending to any given point: also compare the forces tending to any two points round which the same body will revolve through same circle in same period.

7. A body describes a cycloid by action of a force in lines parallel to the base, determine the law of its variation.

8. Find the value of the angular velocity in any curve, and compare the angular velocities of the distance and perpendicular on tangent, and shew that in different similar ellipses, round same force in focus, the angular velocity at same distance from the axis varies as $\frac{1}{(\text{dist.})^{\frac{3}{2}}}$.

9. If any number of bodies revolve round a common centre, and centripetal force $\propto \frac{1}{(\text{dist.})^2}$. The latera recta of the orbits described will be in the duplicate ratio of the areas described in the same time.

10. Determine the law of force tending to the vertex by which a body may describe any of the conic sections.

11. Give a method of comparing the densities of the Earth and planets.

12. A body describes an ellipse uniformly, determine the law of force tending to either focus.

CORPUS CHRISTI COLLEGE, May 1830.

Sections VI. *to* XI.

1. Find the place of a body in a parabolic orbit, corresponding to a given time.

2. Given the time between the nodes of a comet's orbit, find the position of the line of nodes.

3. Determine geometrically and analytically, the times of ascent or descent of a body projected from a given point, towards or from the centre of force with a given velocity.

4. If a body move in the logarithmic spiral round a mass of attracting matter placed in its pole; determine the external and internal spaces due to the velocity at any point.

5. If one body move in a curve, and another ascend or descend in a straight line, acted on by any kind of centripetal force, and their velocities be equal at equal distances from the centre in any one case; their velocities will be equal at all other equal distances.

6. If a body be projected at a given distance from a centre of force, in a given direction and with a velocity equal to that from a finite distance; find the equation to the orbit described. The force varying as $\frac{1}{D^3}$.

7. The force varying as $\frac{1}{D^5}$; shew under what restrictions of the velocity of projection the body's approach towards the centre, and its recess towards infinity, will be limited by asymptotic circles.

8. If a body acted on by a central force which at any distance (r) $= \frac{m}{r^2} + \frac{h^2}{r^3}$, be projected from an apse at a distance from the centre $= \sqrt{\frac{m}{h}}$: find the polar equation to the orbit described, and shew that the time of describing an angle (θ)

$$= \frac{m}{2 \,.\, n \,.\, h^2} \cdot \left(\frac{n \,.\, \theta}{1 + n^2 \,.\, \theta^2} + \tan.^{-1} .\, n\theta \right), \text{ where } n^2 = \tfrac{1}{2} \left(\frac{m^3}{h^5} \right)^{\frac{1}{3}}.$$

9. The difference of the forces at corresponding points in the fixed and revolving orbit varies inversely as the cube of the distance.

10. Explain Newton's method of finding the angle between the apsides in orbits nearly circular, and apply it when the force

$$= m \,.\, r + \frac{m'}{r^3}.$$

11. Find the time of an oscillation in an epicycloid, the body

being acted upon by a repulsive force varying as the distance from the centre of the globe.

12. Determine under what circumstances a body will describe a circle on a surface of revolution, when acted on by forces in a plane passing through the axis.

13. Find the tension of a pendulum moving in a spherical surface.

14. If two bodies acted on only by their mutual attractions be projected in given directions and with given velocities, determine their motions; the law of attraction being the inverse square of the distance.

15. Determine the motions of a system of bodies attracting each other with forces varying directly as the distances between their centres, and also the periodic time of any one of the bodies.

16. Define the central disturbing force on the Moon; find an expression for it, and shew that its mean value $= -\frac{1}{2}$ the mean addititious force.

17. When the force varies as the distance, shew that there will be no disturbance. Had this law pervaded the universe, what would have been the consequence?

18. Why are the principles of the ninth Section inapplicable to the complete explanation of the planetary motions?

19. Find the effects produced by the ablatitious force on the motion of the nodes.

20. Determine under what condition a body will describe an ellipse, when acted upon by three masses of attracting matter; one situated in the centre, whose attractive force is directly as the distance, and one in each focus, the law of whose attraction is the inverse square,

CAIUS COLLEGE, MAY 1830.

1. THE spaces which a body describes, when acted upon by any finite force, are "ipso motus initio" in the duplicate ratio of the times.

2. Find the equations of motion of a body moving in a plane and acted upon by any forces in that plane.

3. A body being acted upon by a central force, the areas described are in one plane, and are proportional to the times.

4. A body moving in any orbit whatever is acted upon by a force tending to a given fixed centre; determine the force.

5. A body is acted upon at every point by a force which is proportional to its distance from a given centre towards which it tends: find the orbit described, and the position and magnitude of its axes.

6. Prove that the force by which a body will describe the opposite hyperbola round the focus is repulsive and varies as $\frac{1}{D^2}$.

7. The velocity in a curve at any point is equal to that generated by the force at that point, continued uniform, and acting on a body while it moves through one-fourth of the chord of curvature drawn at that point through the centre of force.

8. Find the time of a body describing any portion of a parabola about a centre of force in the focus.

9. A body falls from rest from a given point towards a centre of force: required the laws of force for which the time can be found.

Find the time to the centre when the force varies as $\frac{1}{D^2}$.

10. If a body move in a curve, and another body move in a straight line, acted upon by the same central force; and if their velocities be equal at any corresponding equal distances from the centre, they will be equal at any other equal distances.

11. The force varying as D^n, deduce the apsidal equation, and determine the number of possible roots which it will admit.

12. The difference of the forces by which a body may be made to move in the fixed and in the moveable orbit varies as $\frac{1}{D^3}$ from the centre.

13. Find the time of an oscillation in the hypocycloid; the force tending to the centre of the globe and varying as the distance.

14. The inclination of the Moon's orbit to the ecliptic is greatest when the line of nodes is in syzygies, and least when it is in quadratures.

15. Find the mean disturbing force on the Moon in a whole revolution, and compare it with the mean addititious force.

16. Find the attraction of a sphere on a particle placed without it, the law of attraction being

$$mr + \frac{m'}{r^2}.$$

17. Find the horary motion of the nodes in the Moon's orbit, supposed circular.

18. If a system of bodies moving freely be acted upon only by their mutual forces, the sum of each particle multiplied into the projections of the areas described about a fixed point is proportional to the time.

PROBLEMS.

TRINITY COLLEGE, 1823.

1. How much paper 30 inches wide, would be required to hang a room which is 16 feet by 14 feet, and 9 feet high?

2. Calculate the common logarithm of 99 to four places of decimals, the Napierian logarithm of 10 being 2·3026 nearly.

3. Solve the following equations:

(1). $\dfrac{17x+5}{3} - 3x = 18 - \dfrac{11}{2}x.$

(2). $\dfrac{119+10x}{5x-7} = 9x - \dfrac{7x+17}{x+1}.$

(3). $ax^2 + b^2x + c^3 = d^3.$

(4). $8x - 3y = 19 + \dfrac{7x+1}{8}$; and $3x - 5y = 18 - \dfrac{13y-3}{11}.$

(5). $4x^5 - 9x^4 - 11x^3 + 3x^2 + 7x + 6 = 0.$

4. *BD, CD* are perpendiculars to the sides *AB, AC* of the triangle *ABC,* and *CE* is drawn perpendicular to *AD*; prove that the triangles *ABC, ACE* are similar.

5. Prove that the introduction of imaginary quantities does not vitiate an algebraical reasoning.

6. Express the arc whose sine is x, the radius of the circle being a, in terms of an arc of the circle whose radius is unity.

7. Two spectators at given places observe at the same instant a balloon, which rises into the air at an uniform rate; and after a

given interval of time they again observe it: they note down its azimuths and altitudes. From these data find

(1). The two elevations of the balloon.

(2). The rate at which it moves.

(3). The direction of the wind.

8. An equation of n dimensions, in which p successive terms are wanting, cannot have more than $n - p + 1$ real roots.

9.
$$a - \cfrac{c^2}{2a - \cfrac{c^2}{2a - \text{\&c.}}} = c \cdot \frac{\sin.\theta}{\tan.n\theta},$$

if $\cos.\theta = \frac{a}{c}$, and $\frac{c^2}{2a}$ occurs $(n - 1)$ times.

10. Two given circles touch each other internally; find the locus of the centres of all circles which can be drawn to touch both the circles.

11. Deduce the equations

(1). To the surface of a paraboloid.

(2). To the curve of the thread of a screw.

(3). To the curve made up of an equilateral hyperbola, and the circle which touches it at the vertices.

12. If a false die have its angles all right, and two of its opposite sides squares, but their distance, instead of being equal to a side of the square, bearing to it the ratio of n to 1; then will the probability that one of the square faces will turn up be $\frac{\theta}{\pi}$; θ being the arc whose sine is $\frac{1}{1 + n^2}$.

13. Assuming the curve of speediest descent from one given point to another, find the actual time of descent.

14. Differentiate $\frac{x.\log.y}{y.\log.x}$ twice successively, and prove that

$$\frac{d^2.xyz}{xyz} = \frac{dx}{x} \cdot \frac{dy}{y} \cdot \frac{dz}{z}\left(\frac{x}{dx} + \frac{y}{dy} + \frac{z}{dz}\right) + \frac{d^2x}{2x} + \frac{d^2y}{2y} + \frac{d^2z}{2z}.$$

15. Differentials of the form $(a + cx^m)^n x^r dx$ can be made rational, if the three quantities n, $\frac{r+1}{m}$, $n + \frac{r+1}{m}$ are not all fractions.

16. Integrate $\epsilon^{x^2} dx$, and $(a + a_1 x + \cdots + a_n x^n)\, dx^2$.

Solve the equation $\frac{dy}{dx} = \frac{7a^4 - 5x^2y^2}{2a^2x^2}$.

17. Find the curve which has this property, that the perpendicular let fall from a fixed point upon the tangent at any point is a mean proportional between a given line, and the distance of the two points; and consider every curve which solves the differential equation.

18. Sum the following series:

(1). $2 + 5 + 11 + 23 +$ &c. to 10 terms.

(2). $1.3.5 + 2.4.6 +$ &c. to n terms.

(3). $\frac{1}{1.3.5} + \frac{1}{2.4.6} +$ &c. to n terms, by increments.

(4). $\frac{1}{1.2.3} + \frac{1}{4.5.6} + \frac{1}{7.8.9} +$ &c. *in inf.*

(5). $\cdots \frac{1}{2} \cdot \frac{1}{a^{\frac{1}{2}} + 1} + \frac{1}{a+1} + \frac{2}{a^2+1} + \frac{4}{a^4+1} + \cdots$

continued indefinitely both ways.

19. *A* and *B* play with false dice, such that neither *A*'s ace, nor *B*'s size can turn up; they throw together for £10, to be won by the highest, but to be divided equally if the throws are equal: what is *A*'s chance of winning, and what is his chance worth?

20. There are two spheres of equal diameter and weight, the one solid, the other hollow, and the specific gravities of their materials are known, but not accurately: with aid of a comparison of their momenta of rotation discover very accurately their specific gravities, and the thickness of the shell of the hollow sphere.

21. State the error in the estimated motion of the Moon's apogee, by which Astronomers before Clairaut were perplexed, and point out the fallacy from which it arose.

22. The curve of speediest descent from a given point to a given vertical plane, is a cycloid, the axis of which is in the given plane.

TRINITY COLLEGE, 1824.

1. EXPAND $\frac{1}{\sqrt{(a^3 - bx^2)}}$ by the binomial theorem.

2. Required the present value of an estate of £1500 a year, at 3 per cent compound interest.

3. The sides of a plane triangle being a, b, c, prove that the perpendicular on (a) from the opposite angle

$$= \frac{1}{2a} \sqrt{\{2a^2b^2 + 2a^2c^2 + 2b^2c^2 - (a^4 + b^4 + c^4)\}}.$$

4. The product of (n) consecutive integers is divisible by $1 . 2 . 3 \cdots (n)$. Required proof.

5. Let Pp be an ordinate to the diameter AB of a circle, meeting the circumference in P, p: draw AP, Bp intersecting in Q: required the locus of Q.

6. (1). Given $\left.\begin{aligned} x + y &= 5, \\ x^4 + y^4 &= 97; \end{aligned}\right\}$ find (x) and (y).

(2). Solve, by De Moivre's formula, the equation

$$x^5 + 1 = 0.$$

(3). Find the number of positive and of negative roots in the equation

$$x^5 - 5x^4 - 15x^3 + 85x^2 - 26x - 120 = 0.$$

(4). In the equation $x^3 - 4x^2 + x + 6 = 0$. Required the sum of the cubes of the roots.

7. Deduce the equation of equilibrium on the inclined plane, from the principle of the lever.

8. In the ellipse or hyperbola, if lines drawn from the extremities of any diameter to any point in the curve intersect the major axis in angles θ, θ', prove that

$$\tan.\theta . \tan.\theta' = \mp \frac{b^2}{a^2}.$$

9. Determine which is the greatest, and which the least, of the three squares which may be inscribed in a given triangle.

10. If α, β, γ, be the angles which the diagonal (d) of a parallelopiped makes with the edges, a, b, c, prove that

$$d = a \cos.\alpha + b \cos.\beta + c \cos.\gamma.$$

11. The equilibrium of a balance having been disturbed by the addition of a very small weight to one of the equal bodies weighed in it, required the corresponding inclination in the needle of the balance.

12. Required the number of imaginary roots in the equation

$$x^5 - 6x^4 + 5x^3 - 30x^2 + 6x - 36 = 0.$$

13. (1). Explain the method of finding the *absolute* maximum and minimum values of a function of two variables, and deduce the criterion which serves to determine whether the result be a maximum or a minimum.

(2). Amongst all triangular pyramids of given base and altitude, to find that which has the least surface.

14. An ellipsoid being given in space, determine its centre.

15. (1). If a body acted on by a central force describe a polar curve whose equation is $r = \varphi(\omega)$, shew that

$$f = -\frac{h^2}{2dr} \cdot d\left(\frac{1}{r^2} + \frac{dr^2}{r^4 d\omega^2}\right),$$

(h) denoting twice the area described in the unit of time.

(2). Required the value of (f) in lines of the second order.

16. Find from what axis a body must be suspended, that it may oscillate in the least time possible.

17. If to the sine of any arc in a circle, whose radius is (r), there be added the corresponding cosine, with its proper sign, prove that the locus of the points so determined will be an ellipse whose axes are

$$2r\sqrt{[1 + \sqrt{\{1 + \sqrt{(1 + \cdots\cdots)}\}}]}$$
$$2r\sqrt{[1 - \sqrt{\{1 - \sqrt{(1 - \cdots\cdots)}\}}]}.$$

18. Given one principal axis of a system, to find the other two.

19. A cylindrical shell of small thickness unrolls itself from a vertical string which passes over a fixed pulley, and has a weight attached to its extremity; find the tension of the string.

20. Differentiate

(1). $u = e^x \sqrt{\dfrac{1+x}{1-x}}.$

(2). $u = \log. \tan. \dfrac{x}{y}.$

Integrate

(1). $du = \dfrac{dx}{(1+x^2)^3}.$

(2). $du = d\theta \sec.^3\theta.$

(3). $a^2d^2y - ydx^2 = 0.$

21. Let AP, AP', AP'' ··· be *synchronous* arcs of any number of inverted cycloids originating in A, and having their bases in the same horizontal line; required the nature of the curve which passes through the points P, P', P'' ···.

22. A flying bridge is established on a river of known breadth and of uniform current; the cable being fixed at a point equi-distant from the banks, what must be its length, in order that the velocity of the bridge when half way across may be the greatest possible?

23. If the horizontal parallax of the Moon be accurately observed, at the same time, in two different latitudes, shew that the ratio of the equatorial and polar diameters of the Earth may be thence determined.

TRINITY COLLEGE, 1827.

1. DESCRIBE a circle, which shall pass through three given points that are not in the same straight line.

2. A sum of £400 is due in two years, and one of £2,100 in eight months. Find the equated time of one payment, interest being 5 per cent.

3. If from one of the angles of a rectangle a perpendicular be let fall on the diagonal D, and from the point of intersection perpendiculars P, p, be let fall on the sides containing the opposite

angle, then will

$$P^{\frac{2}{3}} + p^{\frac{2}{3}} = D^{\frac{2}{3}}.$$

4. Solve the following equations:

(1). $$\frac{x}{8} + 8y = 194,$$

$$\frac{y}{8} + 8x = 131.$$

(2). $$\frac{ax - b^2}{\sqrt{(ax)} + b} = c + \frac{\sqrt{(ax)} - b}{d}.$$

(3). $$\frac{x^2 + xy + y^2}{x + y} = 7,$$

$$\frac{x^2 - xy + y^2}{x - y} = 9.$$

(4). Find the value of x to 4 places of decimals in the equation $3^{2x} . 2^{3x} = 10000$,

log. 2 being $= \cdot 3010300$,

log. 3 $ = \cdot 4771213$.

5. If the equation $\left\{\begin{array}{l} x^4 + y^4 = b^4 \\ x + y = a \end{array}\right\}$ be solved by means of an intermediate quadratic from which xy is obtained, the negative value in this last equation must always be taken in order to obtain the possible values of x and y.

6. The sum of n quantities in arithmetic progression $= a$, and the sum of their squares $= b^2$; find them.

7. A cask is full of wine and contains (a) gallons: b gallons are drawn from it and the void filled with water: b gallons of the mixture are then drawn from it, and replaced as before by water. How much wine will remain in the cask after the process has been repeated n times?

8. Explain the method of finding in the tables a number corresponding to a given negative logarithm.

9. Having given the side of a regular polygon of n sides, find its area.

10. Sum the following series:

(1). $\frac{1}{1.3} + \frac{1}{2.4} + \frac{1}{3.5}$ + &c. *ad infinitum.*

(2). $1^2 - 2^2 + 3^2 - 4^2 +$ &c. $\pm x^2$.

(3). Shew that

$$x \sin.\theta + \tfrac{1}{2} . x^2 \sin.2\theta + \tfrac{1}{3} . x^3 \sin.3\theta + \&c. \textit{ ad infinitum.}$$

$$= \tan.^{-1} \frac{x \sin.\theta}{1 - x . \cos.\theta}.$$

11. Shew that

$$\{a + b\sqrt{(-1)}\}^{\frac{m}{n}} + \{a - b\sqrt{(-1)}\}^{\frac{m}{n}} = 2(a^2 + b^2)^{\frac{m}{2n}} . \cos. \frac{m}{n}\theta,$$

$$\text{where } \cos.\ \theta = \frac{a}{\sqrt{(a^2 + b^2)}}.$$

12. An impossible root of $x^n - 1 = 0$ is α. Prove that any n consecutive terms of the series

$$\ldots\ldots \alpha^{-3},\ \alpha^{-2},\ \alpha^{-1},\ \alpha^0,\ \alpha^1,\ \alpha^2,\ \&c.$$

are the n roots of the equation.

13. *AB, CD* are two arcs of a circle intercepted between a chord and a given diameter. Determine the position of a chord, so that one arc shall be triple of the other.

14. Let θ be the angle observed at a point in the horizontal plane, between two objects whose altitudes (which are small) are a and b. Then, if $\theta + \delta\theta$ be the reduced horizontal angle, $\delta\theta$ will be nearly equal to

$$\left(\frac{a+b}{2}\right)^2 . \tan. \frac{\theta}{2} - \left(\frac{a-b}{2}\right)^2 . \cot. \frac{\theta}{2}.$$

15. Six persons are to be placed by lot at a circular table: what are the chances against any two assigned friends being seated next each other?

16. Determine the equation and position of the rectilineal and hyperbolic asymptotes of the curve, whose equation is

$$x^3 - 3axy + y^3 = 0.$$

17. Explain what is meant by cusps, by multiple, and conjugate points in a curve. Shew whether such points exist in the curves whose equations are

(1). $ay^2 - x^3 = 0.$

(2). $(y - ax^2)^2 = bx^5.$

(3). $ay^2 - x^3 - bx^2 = 0.$

(4). $(x^2 - a^2)^2 + (y^2 - b^2)^2 = 0.$

18. Integrate the following differentials:

(1). $$\frac{\{1 + \sqrt{x} - \sqrt[3]{(x^2)}\}\,dx}{1 + \sqrt[3]{x}}.$$

(2). $$\frac{x^r\,dx}{x^n - 1}.$$

(3). $$\frac{\varepsilon^x x\,dx}{(1 + x)^2}.$$

19. Determine the equation to the curve in which the radius of curvature is inversely as the abscissa.

20. Find the centre of gravity of the thread of the screw.

21. State the principle of virtual velocities, and by it prove that the centre of gravity of a system of heavy bodies connected together in any invariable manner, is, when they are in equilibrium, at the highest or lowest point.

22. Prove the centres of oscillation and suspension to be convertible: and shew the use of this property in determining practically the length of a pendulum which shall oscillate seconds.

23. Shew how the curve of quickest descent between two given points may be constructed, assuming it to be a cycloid.

TRINITY COLLEGE, MAY 1828.

1. If the square described upon one of the sides of a triangle is equal to the sum of the squares described upon the other two; the angle contained by these two is a right angle.

2. In a given circle to inscribe a triangle equiangular to a given triangle.

3. To draw a straight line perpendicular to a plane from a given point above it.

4. Shew that at any stage of the operation for finding the greatest common measure of two algebraical expressions, a factor may be introduced into or expunged from either divisor or dividend, which does not appear in the other.

5. Prove that for all values of n the two first terms of the expansion of $(1 + x)^n$ are $1 + nx$.

6. Determine in what cases $\sqrt[2m]{-a} \times \sqrt[2m]{-b}$ is possible and in what impossible.

7. Every equation of an odd number of dimensions has at least one real root, and every equation of an even number of dimensions whose last term is negative has at least two real roots, one positive and the other negative.

8. Find the amount and the present worth of an annuity, at compound interest.

9. Given the three sides a, b, c, of a triangle, c being nearly equal to $a + b$. Find the angle C.

10. Investigate a general expression for the tangent of the sum of any number of arcs in terms of the tangents of the arcs.

11. In a spherical triangle

$$\tan. \frac{A+B}{2} = \frac{\cos. \frac{a-b}{2}}{\cos. \frac{a+b}{2}} \cot. \frac{C}{2}.$$

12. Prove Napier's rules when one of the sides is the middle part.

13. The projection of a circle on a plane inclined at any angle to its plane is an ellipse. Prove this, and hence shew that

$$PV.VG : QV^2 :: CP^2 : CD^2.$$

14. The subcontrary section of an oblique cone is a circle.

15. There can be no more than five regular solids.

16. Find directly the differential coefficient of $\cos.x$.

17. If $u = f(x + k)$, $\frac{du}{dx} = \frac{du}{dk}$.

18. Transform the equation to the Lemniscata

$$(x^2 + y^2)^2 = a^2 (x^2 - y^2)$$

from rectangular to polar co-ordinates, and then find its area.

19. Investigate the method of determining the maxima and minima of a function of two variables, and the criterion for distinguishing between them.

20. The moment of the resultant of two forces acting on a lever at the same point is equal to the sum or difference of the moments of the components, according as they tend to turn it the same or different ways.

21. Prove that the times of descent down all chords drawn to the lowest point in a circle are equal. Does this hold when the plane of the circle is not vertical?

22. Investigate the equations of motion in a plane

$$X = \frac{d^2x}{dt^2}, \quad Y = \frac{d^2y}{dt^2}.$$

23. Define the centre of percussion, and deduce the formula for determining it.

24. Define the principal axes of a system, and shew that they are at right angles to each other.

TRINITY COLLEGE, May 1828.

1. Explain fully the rule for the division of one algebraical expression by another, particularly as regards the arrangement of the terms in each.

2. If from the cube of any odd number greater than 1 the number itself be subtracted, the remainder will be divisible by 24.

3. Given two points *A*, *B*, within a circle. Determine the point *P* in the circumference such that the angle *APB* shall be a maximum.

4. On a given base, triangles are described such that the vertical angle is always an arithmetic mean between the angles at the base. Determine the locus of the vertices of the triangles.

5. A given cone is cut by a plane passing through a point in the circumference of the base. Find the position of the plane when the area of the section is a maximum or minimum; distinguish between the two cases, and shew what are the limits of the values of the vertical angle of the cone which admit such a maximum or minimum.

6. APB is a quadrant, AP any arc, PM its sine. BM is drawn intersecting the radius CP in Q. Find the locus of Q.

7. Sum the following series:

(1). $\frac{1}{1.5}+\frac{1}{3.7}+\frac{1}{5.9}+$ &c. to n terms.

(2). $\frac{3}{1.2}\cdot\frac{1}{2}+\frac{4}{2.3}\cdot\frac{1}{2^2}+\frac{5}{3.4}\cdot\frac{1}{2^3}+$ &c. to n terms and *ad infinitum*.

(3). $\cos.^2\alpha+\cos.^2(\alpha+\beta)+\cos.^2(\alpha+2\beta)+$ &c. to n terms.

Shew that the sum of the third series, when $n\beta=2\pi$, is independent of the value of α.

8. Three persons A, B, and C, agree to throw a single die successively till one shall have thrown an ace. Compare their respective chances.

9. From a bag containing $2n+1$ balls, $2n$ are taken one by one so that the 1st, 3rd, 5th... $(2n-1)$th balls are white, and the 2nd, 4th, 6th... $2n$th red. Shew that the chance that the remaining ball is white = the chance that it is red, and find the chance that it is neither.

10. A given beam rests on a fixed point, and with its end against a curve such that there is an equilibrium in all positions of the beam. Determine the curve.

11. Integrate

(1). $\frac{x^2dx}{\{x-\sqrt{(x^2-1)}\}^3}$, (2). $\frac{(\sec.\theta+\tan.\theta)\,d\theta}{\sec.\theta-\tan.\theta}$,

(3). $\frac{\cos.x\,dx}{a\sin.x-b\cos.x}$.

12. Determine the relations of x and y in the following equations:

$$(1).\ a\,dx + (x + y)(dx - dy) = 0.$$

$$(2).\ \frac{d^2y}{dx^2} + 2x \cdot \frac{dy}{dx} - (a - y)\,x^2 = 0.$$

13. From successive points in a horizontal line a ball of given elasticity is let fall on a given inclined plane. Find the equation to the boundary of the parabolas described.

14. A perfectly flexible chain of given length, being placed in a given position on the circumference of a circle whose plane is vertical, slides off down a tangent to the circle at the point which the lower end of the chain originally reaches. Required its velocity in any given position.

15. Determine the equation to the curve symmetrical to its axis, such that, when suspended by its vertex, the time of a small oscillation in its own plane of any portion of the curvilinear area cut off by a line perpendicular to the axis shall be constant.

16. A bullet is placed in a straight tube which is made to revolve uniformly in a vertical plane. Determine the motion of the bullet.

TRINITY COLLEGE, JAN. 9, 1829.

1. WHEN a body is in equilibrio, its centre of gravity is at its highest or lowest point.

2. Find the greatest height of a projectile above a given inclined plane, passing through the point of projection.

3. Find the moment of inertia of a ⊙ revolving about its diameter.

4. What is meant by the *sensibility* and *stability* of a balance: and on what do they severally depend?

5. If a be the height of the fixed valve of the common pump above the water, b the length of the stroke of the piston, and h the

height of a column of water in equilibrio with the atmosphere, the water will not rise above the piston unless b be greater than $\frac{(a+b)^2}{4h}$.

6. Explain the principle of the hydrometer.

7. In the Astronomical telescope, the magnifying power is increased by increasing the convexity of the eye-glass: within what limits is this increase confined?

8. The limb of a quadrant is divided into spaces of 5° each, and eleven divisions of the limb correspond to ten divisions of the vernier: to what accuracy will the instrument read off?

9. If a small pencil of homogeneous light be refracted into a sphere, to find the $\angle$ of incidence, so that the rays may emerge parallel after any number of reflections within the sphere.

10. Explain the necessity and reason of the Gregorian correction of the calendar.

11. Is the aberration of the moon a *sensible* quantity?

12. Find the precession in right ascension.

13. In what manner do we determine the altitude of the Sun, Moon, or other heavenly body at sea by means of the sextant? How is the *meridian* altitude determined?

14. Find the law of force tending to the focus of the hyperbola.

15. When the force varies as $\frac{bA^m + cA^n}{A^3}$, find the angle between the apsides.

16. Make a body oscillate in a hypocycloid.

17. When a body revolves on an axis and a force is impressed, tending to make it revolve upon another, it will revolve on neither, but on a line in the same plane with them, dividing the angle which they contain, so that the sines of the parts are in the inverse ratio of the angular velocities with which the body would have revolved about the said axes separately.

18. The increase of weight in going from the equator to the pole $\propto \sin.^2$ latitude nearly.

19. Explain Newton's method of determining the ratio between the polar and equatorial diameters of the Earth.

20. Explain generally the cause of Neap and Spring Tides.

21. What is the use of the fly-wheel in machinery?

22. Let any forces act upon a system moving round a fixed axis; to find the pressure on the axis.

TRINITY COLLEGE, JAN. 1829.

1. EXPLAIN what is meant by the locus of a point. By the intersection of two loci, solve the following problems:

(1). To describe a ⊙ which shall pass through three given points.

(2). To find a point at which three given $|^s$ placed contiguously, and in the same $|$, shall subtend equal $\angle^s$.

State the circumstances under which the problems are impossible.

2. Find the length of the $\perp$ let fall from a given point on a given straight line.

3. Reduce the following fractions:

(1). $\dfrac{a^3 - 4a^2b + 4ab^2 - b^3}{2a^2 - 6ab + 4b^2}$ to its lowest terms.

(2). $\dfrac{1}{\sqrt{a} + \sqrt{b} + \sqrt{c}}$ to one having a rational denominator.

4. Solve the following equations:

$$(1).\ \left.\begin{aligned} \frac{1}{x} + \frac{1}{y} &= \tfrac{1}{2}, \\ \frac{2}{xy} &= \tfrac{1}{9}. \end{aligned}\right\}$$

$$(2).\ \frac{a - \sqrt{(a^2 - x^2)}}{a + \sqrt{(a^2 - x^2)}} = \frac{m}{n}.$$

$$(3).\ x^6 - a^6 = 0.$$

5. If $\sqrt{a}$ be a real root of Des Cartes' reducing cubic, the roots of the biquadratic are

$$\frac{\sqrt{a}}{2} \mp \sqrt{\left(-\frac{a}{4} - \frac{q}{2} - \frac{r}{2\sqrt{a}}\right)},$$

$$\text{and} \qquad -\frac{\sqrt{a}}{2} \pm \sqrt{\left(-\frac{a}{4} - \frac{q}{2} + \frac{r}{2\sqrt{a}}\right)}.$$

6. The angle observed at a point in the horizontal plane between two objects whose altitudes (which are small) are a, b, and θ. Shew that if $\theta + \delta\theta$ be the reduced horizontal $\angle$, $\delta\theta$ will be nearly equal to

$$\left(\frac{a+b}{2}\right)^2 \tan.\frac{\theta}{2} - \left(\frac{a-b}{2}\right)^2 \cot.\frac{\theta}{2}.$$

7. What are meant by conjugate points in a curve? What by cusps? Do such points exist in the curves whose equations are

$$(1).\ (x^2 - a^2)^2 + (y^2 - b^2)^2 = 0.$$

$$(2).\ (y - x)^2 = x^9.$$

$$(3).\ y = ax^2 \pm bx^{\frac{5}{2}}.$$

8. Find the rectilineal and hyperbolic asymptotes of the curve whose equation is

$$x^3 - 3axy + y^3 = 0.$$

9. Having given the inclination of two planes to each of the three rectangular co-ordinate planes, find their inclination to each other.

10. Find the length of the common parabola, and integrate the differentials

$$(1.)\ \frac{dx}{\sqrt{(a + bx - cx^2)}}.$$

$$(2.)\ \frac{dx}{\sqrt{(1 - x^4)}}.$$

11. Integrate the equation $\frac{d^2y}{dx^2} + P.\frac{dy}{dx} + Qy = R$, where P, Q, R are functions of x only.

12. If three forces keep a point at rest, they are to each other as the sines of the angles contained by the other two.

13. Find the centre of gravity of the figure which is the difference of two isosceles $\triangle^s$, on the same base, and on the same side of it.

14. Assuming the principle of virtual velocities, shew that the centre of gravity of any number of heavy bodies connected together in an invariable manner is, when they are at rest, at the highest or lowest points.

15. Find the law of thickness of a string, so that it may hang in the form of a parabola with its axis vertical.

16. The reflecting curve is the reciprocal spiral, and the radiating point is situated in the pole. Find the equation to the caustic.

17. Given the height of a vertical style, the zenith distance of the Sun's centre, and its apparent diameter; find

(1). The length of the shadow of the style.

(2). The breadth of the penumbra.

(3). The distance of the centre of the penumbra from the point corresponding to the Sun's centre.

18. A given cylinder full of water and closed at the top, is whirled round its axis with a given uniform velocity; find the pressure on the lid.

19. A particle of matter is situated in the axis of a paraboloid, consisting of equal particles, each of which attracts with a force proportional to the distance; find the magnitude and position of a sphere which shall attract it with an equal force.

20. A body moves on a rigid logarithmic spiral, the force, which varies inversely as the nth power of the distance, being situated in the pole: find the pressure at any point, and where, and under what circumstances, the body will quit the spiral.

21. At a given time to find the inclination of the Lunar orbit to the plane of the ecliptic [*Newton*, Book III. Prop. 35.]

TRINITY COLLEGE, JUNE 1829.

1. DESCRIBE an isosceles triangle having each of the angles at the base double of the third angle.

2. Find the interest and discount of £500 for 7 months at $3\frac{1}{2}$ per cent per ann. simple interest.

3. Find the value of $\sqrt[3]{\left\{-2+\frac{10}{\sqrt{27}}\right\}}$ to two places of decimals.

4. Prove the binomial theorem when the index is a positive integer, and write down the nth term of the expansion of $\frac{1}{\sqrt{(1-x^2)}}$.

5. Determine the commensurable roots of the equation

$$x^4 - 6x^3 - 27x^2 - 4x + 36 = 0.$$

6. If the roots of the equation

$$x^n - px^{n-1} + qx^{n-2} - \&c. = 0,$$

are all possible, the roots of the equation

$$nx^{n-1} - (n-1)px^{n-2} + (n-2).qx^{n-3} - \&c. = 0,$$

lie between those of the former. If the former equation has impossible roots, an odd number of roots of the latter lie between each of those of the former.

7. Shew how the sine of 1′ may be found.

8. Determine the distance between two visible but inaccessible objects on a horizontal plane.

9. If A, B, C be the angles of a spherical triangle, and S the surface of the whole sphere, the area of the triangle

$$= \frac{S}{4} \cdot \frac{A^\circ + B^\circ + C^\circ - 180^\circ}{180^\circ}.$$

10. Also if a, b, c, be the sides opposite to the angles A, B, C,

$$\tan.\frac{A+B}{2} = \frac{\cos.\frac{a-b}{2}}{\cos.\frac{a+b}{2}}.\cot.\frac{C}{2}.$$

11. Prove that in an ellipse $SP \times HP = CD^2$.

12. Expand a^x in a series proceeding by powers of (x).

13. Having given the equation to a straight line, find that of another line which is perpendicular to the former, and passes through a given point.

14. Find the rectangular equation to the conchoid of Nicomedes, and from it deduce the form of the curve, shewing in what cases there will be a nodus, cusp, or conjugate point.

15. If the forces P, Q, &c. balance the forces P', Q', &c. acting upon the arms of a lever, shew that

$$P.CM + Q.CN + \&c. = P'.CM' + Q'.CN' + \&c.$$

where CM, CN, &c. and CM', CN', &c. are the perpendiculars from the fulcrum C on the directions of the forces. Also if C be made the origin of rectangular co-ordinates, and P's direction make an angle (α) with the axis of (x), and x, y, be the co-ordinates of any point in P's direction, prove that $CM = y \cos.\alpha - x \sin.\alpha$.

16. State and illustrate the second law of motion, and mention several facts from which its truth may be inferred.

17. Find the time of oscillation in a small circular arc.

18. State the principle of virtual velocities; and prove it either generally, or in the case where three forces act upon a point.

19. Draw a tangent to an ellipse from a given point without it.

20. Prove that $\tan.^{-1} . x = x - \frac{x^3}{3} + \frac{x^5}{5} - \&c.$

21. Sum the series $1^3 + 2^3 + 3^3 + \&c. \cdots$ to (n) terms.

22. Find the moment of inertia of a semicircle about its diameter.

23. Find the equations to a straight line in space; and determine the angle which two lines, whose equations are given, make with each other.

24. Investigate the general equation of equilibrium of any fluid; and shew from the equation that the resultant of the forces at any point in the surface of a fluid incompressible and perfectly free is a normal to the surface.

TRINITY COLLEGE, June 1829.

1. An equilateral triangle and an equilateral and equiangular pentagon have the same perimeter; compare the radii of the circles inscribed in them.

2. If a straight line be perpendicular to a plane, the projection of that straight line on any other plane and the intersection of the two planes are at right angles to each other.

3. If C be the centre and P any point in the arc of an ellipse, find the angle contained between CP and the normal at P; and determine the point P where that angle is the greatest.

4. Shew that, if $A + Bx + Cx^2 = a + bx + cx^2$, for three particular values of (x), A must $= a$, $B = b$, $C = c$.

5. What must be the form of (n),

(1). that $\frac{n(n-1)}{2}$ may be even, (2). that it may be odd.

6. There are three casks, each of which contain a mixture of water, wine, and brandy in a given ratio; what quantity must be taken from each to form a fourth mixture, which shall contain a given quantity of the above ingredients?

7. If the terms of an equation be so arranged as to be alternately positive and negative; and if A, A', A'', be the coefficients of the 1st, 3rd, 5th ... B, B', B'', of the 2nd, 4th, 6th terms so arranged, and the index of x, in each odd term exceed that in the following even one; shew that the greatest of the ratios

$$\frac{B}{A}, \quad \frac{B'}{A'}, \quad \frac{B''}{A''},$$

is greater than the greatest positive root. Apply this to determine a quantity greater than the greatest positive root of the equation

$$x^5 - 13x^4 + 67x^3 - 171x^2 + 216x - 108 = 0.$$

8. If A and B be two fixed points, and P a point in any curve, such that $AP + BP$ is a "minimum;" shew that AP and BP make equal angles with the normal at P.

9. Calculate the Naperian and common logarithm of 11, each to five places of decimals;

$$\text{Naperian log.}2 = \cdot 6931472$$

$$\text{Naperian log.}5 = 1\cdot 6094379.$$

10. Trace the curve whose equation is $x^2y - a^2x + b^3 = 0$, and draw its rectilinear and hyperbolic asymptotes.

Shew also that the whole area intercepted between the curve and the hyperbolic asymptote, and corresponding to the positive ordinate in the curve, is $= a^2$.

11. If $z = f(x, y)$ be the equation to a curve surface referred to three rectangular axes, and (u) the volume included under a portion of the surface, prove that $\frac{d^2u}{dxdy} = z$; and apply this expression to

find the volume cut off by the planes of xy, yz, and a plane parallel to that of yz, in the surface whose equation is $x^2 + y^2 + az = b^2$.

12. A bag contains 16 balls, 7 of which are white; if 5 are taken indiscriminately, what is the probability of taking three white ones?

13. How does it come to pass, that what appears sufficiently strong in a model, is frequently very weak when executed in large dimensions according to the model? Illustrate this by the consideration of two similar prismatic beams fixed in a wall, acted upon only by their own weight.

14. If two lines SP, HP, revolve about two fixed points S, H, so that $SP \times HP = n^2 . CS^2$, ($C$ being the middle point of SH), find the equation to the curve of which P is the locus; and determine its form according as (n) is greater than, equal to, or less than unity.

15. Draw a plane which shall touch two given spheres, and pass through a given point.

16. Sum the following series:

$$\left.\begin{array}{l} 4 + 7 + 13 + 25 + \\ 1.2 - 2.3 + 3.4 - \end{array}\right\} \text{each to } (x) \text{ terms.}$$

17. Prove that

$$\int \frac{\tan.\theta d\theta}{\sqrt{(\cos.^2\alpha + \tan.^2\theta)}} \begin{pmatrix} \theta = 0 \\ \theta = \frac{\pi}{2} \end{pmatrix} = \frac{\alpha}{\sin.\alpha};$$

and find the relation between (u) and (θ), when

$$\frac{d^2u}{d\theta^2} + u = A \cos.m\theta,$$

and from the general solution deduce the result when $m = 1$.

18. Determine the curve which shall cut a series of circles which touch each other at the same point, so that the tangents to the circles at the points of intersection shall pass through a given point.

19. Having given a solution of a proposed differential equation, determine whether it is comprised in the complete integral or not.

20. A uniform ladder is placed in a vertical plane against an upright wall, and a slight motion is communicated to it, so as just to move it out of the perpendicular; find the time of its falling through a given angle.

21. A given triangle of uniform density, and moveable about the perpendicular from one of its angular points on the opposite side as an axis, is struck by a given force applied at its centre of gravity; find the pressure produced on the axis, and the angular velocity communicated to the triangle.

22. What is the common theory of the arch? Why is it imperfect in a practical point of view? Explain how the true theory is connected with that of roofs, and why the pointed arch must carry a heavy load on its vertex.

23. Explain the physical cause of solar precession and solar nutation; and determine the angular velocity generated by the force of the Sun on the Earth's surface considered a homogeneous spheroid, the integrals of x^2dm, y^2dm, and $xydm$ for the whole spheroid being given.

TRINITY COLLEGE, May 1830.

1. Upon a given straight line to describe a segment of a circle containing an angle equal to a given rectilineal angle.

2. Reduce the fraction $\dfrac{3x^2 - 2xy - y^2}{4x^3 - 2x^2y - 3xy^2 + y^3}$ to its lowest terms.

3. Find the interest on £57. 13*s*. for 9 months at $2\frac{1}{2}$ per cent.

4. Give a method of approximating to the *n*th root of a number, and apply it to the cube root of 1010.

5. Give Cardan's solution of a cubic equation, and shew that it fails unless two roots be impossible, or two roots equal.

6. Resolve $x^6 + 1$ into its quadratic factors.

7. In a plane triangle having given two sides and the included angle, determine the remaining angles.

8. $(\cos.a + \sqrt{-1}\sin.a)^n = \cos.na + \sqrt{-1}\sin.na$. Prove this, whether ($n$) be whole or fractional.

9. Having given the sides of a spherical triangle, determine the sine of half one of the angles, and adapt the formula to logarithmic computation.

10. If C be the right angle in a right-angled spherical triangle, find the variation of the side (a) corresponding to a small variation δc in (c); and shew that when $c = 90°$ the variation is $-\dfrac{\tan.A(\delta c)^2}{2}$.

11. Determine the length of a perpendicular from a given point on a line whose equation is given.

12. Determine the position of the centre and magnitude of the axes of the ellipse whose equation is $9y^2 + 4x^2 + 18y - 16x - 47 = 0$.

13. State the principal steps in the reasoning by which it may be shewn (independently of the differential calculus) that if $f(x)$ be any function of (x), and x be increased by h, $f(x + h)$ may be expanded in a series of the form $f(x) + Ph + Qh^2 + \cdots$

14. Determine the differential of a solid of revolution, and apply it to find the content of a sphere.

15. Integrate the differentials

$$\frac{dx}{\sqrt{(a^2 + x^2)}},\quad \frac{d\,\theta}{\cos.^3\theta},\quad \frac{x^n dx}{\sqrt{(a^2 - x^2)}}\ \begin{pmatrix} x = 0 \\ x = a \end{pmatrix}.$$

16. Determine the co-ordinates of the centre of gravity of an area, which is terminated one way by the axis of (x).

17. State the third law of motion, and the nature of experiments by which its truth may be inferred.

18. Determine the path described by a projectile in vacuo, acted on by gravity, and the place of the body at the end of an assigned time.

19. Prove that

$$\log.n = \log.m + 2\left\{\frac{n-m}{n+m} + \tfrac{1}{3}\left(\frac{n-m}{n+m}\right)^3 + \tfrac{1}{5}\left(\frac{n-m}{n+m}\right)^5 + \cdots\right\}.$$

How should we make use of this series to determine the common logarithms of all numbers from 2 to 18?

20. Deduce the equations (1) to a plane (2) to the curve of the thread of a screw.

21. Determine the motion of weights on a wheel and axle, and the pressure on the axis: taking into account the inertia of the machine.

22. A body moves on an inverted cycloid, with a vertical axis, in a medium the resistance of which $= k \times$ velocity, and is acted on by gravity. Determine the time of an oscillation.

TRINITY COLLEGE, MAY 1830.

1. WHAT is the price of carpeting a room 18 feet 9 inches long, and 12 feet 6 inches broad, at *5s.* a yard?

2. In how many years will the population of a country double itself, if the annual increase be $\frac{1}{140}$th?

log. 2 + ·301030, log. 140 = 2·146128, log. 141 = 2·149219.

3. Solve the equations

$$(1).\ \sqrt[4]{(a+x)} + \sqrt[4]{(a-x)} = b$$

$$(2).\ 13x + 9y = 250,$$

determining all the positive and integer values of x and y, which satisfy the last.

4. State the best method, with which you are acquainted, of determining all the rational roots of a given equation; and apply it to determine those of the equation

$$2x^4 + x^3 - 10x^2 - 2x + 12 = 0.$$

5. In approximating to the value of a quantity by the method of continued fractions, each converging fraction differs from the true value less than any more simple fraction:

6. If ρ be the radius of curvature at a point in an ellipse where the normal makes an angle θ with the axis-major,

$$\rho = \frac{a^2 b^2}{(a^2 \cos.^2\theta + b^2 \sin.^2\theta)^{\frac{3}{2}}}.$$

7. Bisect a cone by a plane passing through the circumference of the base.

8. Shew that if $u = \sqrt{\{(x-a)^2 + (y-b)^2 + (z-c)^2\}}$,

$$\frac{d^2 \cdot \frac{1}{u}}{dx^2} + \frac{d^2 \cdot \frac{1}{u}}{dy^2} + \frac{d^2 \cdot \frac{1}{u}}{dz^2} = 0.$$

9. Apply the theory of maxima and minima to determine the shortest straight line from a point x', y', z', to a sphere whose equation is

$$x^2 + y^2 + z^2 = r^2.$$

10. A bag contains twelve balls of four different kinds, three of each kind; what is the probability of taking one of each sort, if four be taken out at once?

11. A given beam rests with one end on a horizontal plane, and the other on the convex surface of a hemisphere, whose centre is in the plane; determine the magnitude of the horizontal force which must be applied to keep the beam in a given position, and the pressure on the sphere and plane.

12. Determine the locus of the intersections of the tangents of a circle with the perpendiculars upon them from a given point.

13. Two bodies set off at the same time from opposite extremities of a diameter of a sphere, and describe two great circles inclined at a given angle with given uniform velocities; determine their places when nearest to each other.

14. Determine the parabola which shall have a contact of the second order with an ellipse at a given point, and have its axis parallel to the axis-major of the ellipse.

15. Compare the weights of two spheres of equal diameters, the density of the particles in one sphere being uniform throughout, and in the other varying as the *n*th power of their distance from the centre.

16. Determine the *n*th term of the series 11, 15, 21, 29, &c. and the sum of n terms; and find the value of x log. x, when $x = 0$.

17. Integrate the differential equations

(1). $(1 - x^2)\, dy + xydx = adx$,

(2). $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$, where P, Q, R, are functions of (x).

18. Determine the angles which the normal to the curve surface whose equation is $Xdx + Ydy + Zdz = 0$, makes with the axes of co-ordinates.

19. Two bodies, P and Q, connected by a rod (without weight), slide the one on a horizontal plane, the other on a given inclined plane, acted on by gravity; determine the motion.

20. Enunciate the general principle of virtual velocities, and show how its combination with D'Alembert's principle reduces all dynamical problems to the solution of a statical question.

21. A quadrant of a circle, moveable about one of its extreme radii, as an axis, is struck by a given force applied at a given point in a direction perpendicular to its plane: determine the pressure produced on the axis, and the angular velocity communicated to the quadrant.

22. Given the length of a curve, to find its form, that its centre of gravity may be the lowest possible.

ST. JOHN'S COLLEGE, MAY 1828.

1. IN the expansion of $(a + b)^n$, the coefficients taken at equal distances from the beginning and end are equal.

2. Extract the square root of $41 - 24\sqrt{2}$.

3. In a plane triangle, the sum of the sides is to their difference, as the tangent of the semi-sum of the angles at the base, to the tangent of their semi-difference.

4. An equation, which has but one change of signs, can have but one positive root.

5. Prove that the position of the centre of gravity of a plane curve, referred to rectangular co-ordinates x, y, and bounded by the axis of x, is determined by the equations

$$X = \frac{\int xydx}{\int ydx}, \text{ and } Y = \frac{1}{2}\frac{\int y^2dx}{\int ydx}.$$

6. A body upon an horizontal plane will be supported or not, according as the vertical drawn through the centre of gravity falls within, or without the base.

7. Perpendiculars drawn from the foci upon the tangent to an hyperbola, meet it in the circumference of the circle described upon the axis-major as diameter.

8. If four magnitudes of the same kind are proportionals, the greatest and least of them together are greater than the other two together. [*Eucl.* v. 25.]

9. In the ellipse, prove that

$$CP^2 + CD^2 = AC^2 + BC^2.$$

10. If fluids of different specific gravities communicate through a bent tube, the heights above the common surface will be inversely as the specific gravities.

11. The defects of the mercury in the gage of an air-pump after successive strokes are in geometrical progression.

12. To separate $x^n + 1$ into its factors, where n is odd; and to shew that the factors obtained from the general expression will recur after a certain number.

13. Prove the following theorem in finite differences:

$$u_{x+n} = u_x + n\Delta u_x + n \cdot \frac{n-1}{2} \Delta^2 u_x + \&c. + \Delta^n u_x.$$

14. Find the area of a spherical triangle in terms of its sides.

15. Prove that $\frac{p}{q}$ a fraction converging to the value of $\frac{m}{n}$ differs from it by less than $\frac{1}{q^2}$.

16. Find the chance of throwing an ace in three trials with a single die.

17. Give the general equation to a conic section; find the position of the rectangular axes, and determine whether it has a centre or asymptotes, and their position.

18. If a body float in a fluid, explain what is meant by the Metacentre, and find the time of its small oscillations.

19. If the Earth be a spheroid of equilibrium, the force of gravity at any place varies as the normal.

20. Given A, B, and C the moments of inertia of a solid about the three principal axes passing through a given point, prove that for an axis passing through the same point, and making angles α, β, γ respectively with the former, the moment of inertia is equal to

$$A . \cos.^2\alpha + B . \cos.^2\beta + C . \cos.^2\gamma.$$

ST. JOHN'S COLLEGE, JUNE 1829.

1. ADD together ·0125 of a pound, ·0625 of a shilling, and ·5 of a penny, and reduce 11*s*. 9½*d*. to the decimal of a pound.

2. Prove the rule for finding the greatest common measure of two quantities, and reduce $\dfrac{a^6 - b^6}{a^5 - b^5 + a^2b^2 (a - b)}$ to its lowest terms.

3. Extract the square roots of 654481, $1 + \sqrt{-24}$, and the cube roots of 389017, and $2 + 11\sqrt{-1}$.

4. Solve the following equations:

$$(1). \quad \frac{1}{x-2} - \frac{1}{x-4} = \frac{1}{x-6} - \frac{1}{x-8},$$

$$(2). \quad \frac{x}{3} = \frac{2}{x-1}.$$

$$(3). \quad 3x^{\frac{3}{4}} - x^{-\frac{3}{4}} + 2 = 0.$$

$$(4). \quad \left.\begin{aligned} 9x + 8y &= 43 \\ 8x - 9y &= 6 \end{aligned}\right\}.$$

$$(5). \quad \left.\begin{aligned} y^2 + x &= 13 \\ y^4 + x^2 &= 97 \end{aligned}\right\}.$$

5. If $a : b :: c : d$, shew that $a + b : a - b :: c + d : c - d$.

6. Prove the rule for finding the sum of a geometric series, and apply it to $\frac{3}{2} + \frac{2}{3} + \cdots\cdots$ *ad inf.*

7. The three interior angles of every triangle are together equal to two right angles.

8. If the square described upon one of the sides of a triangle be equal to the sum of the squares described upon the other two sides of it; the angle contained by these two sides is a right angle.

9. One circumference of a circle cannot cut another in more than two points.

10. The angles in the same segment of a circle are equal to one another.

11. Inscribe a circle in a given triangle.

12. Ratios that are the same to the same ratio, are the same to one another.

13. Equal parallelograms, which have one angle of the one equal to one angle of the other, have their sides about the equal angles reciprocally proportional.

14. In a plane triangle, of which the sides are a, b, c; and opposite angles A, B, C, prove that

$$\frac{\sin.A}{\sin.B} = \frac{a}{b}, \quad \cos.A = \frac{b^2 + c^2 - a^2}{2bc}.$$

15. Draw a straight line perpendicular to a plane from a given point without it.

ST. JOHN'S COLLEGE, MAY 1830.

1. TRIANGLES and parallelograms of the same altitude are to one another as their bases.

2. If a solid angle be contained by three plane angles, any two of them are greater than the third.

3. Find x and y from the equations

$$(x + y)^{-2} - (x - y)^{-2} = a^{-2} - b^{-2}, \quad (x + y)^2 a^{-2} + (x - y)^2 b^{-2} = 1.$$

Shew that if $a > b$, then $\frac{a + m}{b + m} < \frac{a}{b}$.

4. Given the first term and common difference of an arithmetic progression, find the sum of (n) terms, and the sum of their squares.

5. If m and every greater number, when substituted for x in an equation, give positive results, m is greater than the greatest root.

6. Every equation has at least as many changes of sign as it has positive roots, and as many continuations of sign as it has negative roots.

7. Solve the equations,

$$8x^3 - 6x^2 - 3x + 1 = 0,\ 2^{\frac{4}{3}}x - 2^{\frac{1}{3}}a - 3a^{\frac{2}{3}}x^{\frac{1}{3}} = 0,$$

$$x^3 + 3x^2 - 8x + 10 = 0,\ x^3 - x^2 + 3x + 5 = 0;$$

of which the first has roots in harmonical progression, the second equal roots, the third a root an integer, the fourth imaginary roots.

8. Find all the positive integral values of x and y in the equation $13x - 17y = 54$. Shew how to solve the equation $x^2 - ny^2 = 1$.

9. Find the sines of the sum, and difference of two arcs in terms of the sines and cosines of the arcs themselves. Shew how the sine of $1''$ may be found.

10. In the parabola, prove that the subnormal equals half the latus rectum, and that $4SP \,.\, PV = QV^2$. In the ellipse, shew that $SP \,.\, HP = CD^2$.

11. If a point be kept at rest by three forces acting at the same time, any three lines which are in the directions of the forces, and form a triangle, will represent them.

12. Time of oscillation of a pendulum length $(l) = \pi \sqrt{\frac{l}{g}}$.

13. Define the specific gravity, and density of a body; and if V, W, Q, D, S represent its volume, weight, quantity of matter, density, and specific gravity; write down the equations which connect these quantities.

14. Investigate the equations of equilibrium of any fluid; and shew that the resultant of the forces at any point in the surface of a fluid incompressible and perfectly free, is a normal to the surface.

15. A solid body moveable about a fixed axis is acted upon in all its points by given accelerating forces in planes perpendicular to that axis; prove that the effective accelerating force of rotation at distances (1) from the axis, equals the moment of resultant of the impressed forces $\div$ moment of inertia of the body, both moments being relative to the fixed axis.

16. If a solid body be struck in a direction perpendicular to the plane through its centre of gravity, and an axis about which it is moveable; find the pressure which the axis sustains at the moment of impact, and the conditions in order that it may vanish.

17. What is a generating function? Find those of x and of $\frac{x}{1+x}$. Express u_{x+n} in terms of u_x, and its successive differences.

18. Find the integrals of

$$x^2(-1)^x;\ x^3,\ \frac{x}{(x+1)(x+2)(x+3)},\ \frac{1}{2^x}\log.\tan.2^x\theta;$$

and the value of the continued fraction

$$\frac{a}{a+}\,\frac{c}{a+}\,\frac{c}{a+}\dots\dots \text{ to } x \text{ terms.}$$

19. Find the general equation to surfaces of revolution, and draw the shortest line between two given points on a surface of revolution.

20. In a parcel of (m) cards, is a sequence of (a) hearts; after shuffling the whole parcel, what is the probability that among the first (p) cards turned up, all the hearts are turned up in order?

QUEEN'S COLLEGE, 1826.

1. Describe a circle which shall touch a given circle, and each of two given straight lines.

2. The product of two quantities divided by their greatest common measure, is their least common multiple.

3. Four times the product of the sines of the angles of any triangle, is equal to the sum of the sines of the doubles of those angles.

4. Give the method of solution of a cubic equation by means of trigonometrical tables and formulæ, and explain the circumstances under which this method fails.

5. If there be two lines whose equations are respectively

$$y = ax + b \text{ and } y' = a'x' + b';$$

it is required to find the tangent of the angle between them.

6. Find the equation to a curve generated in the same manner as the Cissoid of Diocles, that part of a parabola which is cut off by the latus rectum being substituted for the generating semicircle.

7. A lever consisting of two equal straight rods at right angles to each other is moveable about the angular point, a given weight is suspended at one extremity, and a variable one at the other; in what manner must the circular arc described by the given weight be graduated, that the amount of the variable weight may be always indicated?

8. A perfectly elastic ball A impinges on another B at rest, and they meet again at the point C after B has been reflected by a vertical plane at D. The ball A, and the distances BD, BC, are given, to find the ball B.

9. A chain of given length is hung over a fixed pulley, at the beginning of the motion the excess of the longer side above the shorter is given; find the time of running off.

10. Find the centre of oscillation of a circular area vibrating in its own plane about a given axis.

11. A body falls down the axis of a cylindrical tube of given radius, inconsiderable thickness, and infinite length, the particles of which attract with forces varying inversely as the square of the distance; find the velocity acquired in falling through a given space.

12. If a small quantity of air be left in the tube of a barometet, what will be the consequent depression of the mercury below the standard altitude?

13. If a solid of revolution move in a fluid in the direction of its axis, compare the resistance on the surface with that on the base; and apply the general formula when investigated to the case of the hemisphere.

14. A cubical vessel of given dimensions has a vertical crevice of given uniform width from top to bottom of one of its sides; required the time in which the vessel would be emptied through that crevice.

15. If two rays inclined to each other at a small given angle fall nearly perpendicularly on a convex mirror of given radius, and placed at a given distance from the point of intersection of the incident rays; what will be their mutual inclination after reflexion?

16. If a ray refracted into a sphere emerge from it after a given number of reflections, determine the deviation of the ray, and the angle contained between the directions in which it is incident and emergent.

17. What is the nature of the caustic when the reflecting surface is a sphere, and the focus of incident rays is in its surface?

18. Find the law of force parallel to the axis, by which a body would be made to describe a cycloid.

19. The corner of a rectangular piece of paper being doubled down, so that the triangle shall always be of a given area; prove that the vertex of the triangle will trace out a lemniscata, whose area equals the area of the triangle, and which may be described by a force placed at its *knot* varying as $\frac{1}{D^7}$.

20. Force $\propto \frac{1}{D^2}$; shew that the velocity at any point of the descent $\propto \tan. \frac{\theta}{2}$; θ being the circular arc whose diameter is the first distance and versed sine the space described.

21. Compare the quantity of light from the Sun which falls on a given area at the pole of the Earth on the longest day, with the quantity which falls on an equal area at the equator on the day in which the Sun's declination is nothing.

22. Investigate an expression for the tangential ablatitious force on P [*Newton,* Sect. 11]. Find when it is a maximum; and find the velocity of P corresponding to this maximum value, V being the velocity of P at quadrature.

23. Find generally the equation to the orbit in fixed space [*Newton,* Sect. 9], and from that equation shew that the difference of force in the fixed and moveable orbit varies as $\frac{1}{D^3}$.

24. Explain the construction of Hadley's Sextant.

25. Give the latitude of a place, and the declination of the Sun; find the time that the Sun is above the horizon.

26. Shew how the time, duration and magnitude of a lunar eclipse may be computed.

QUEEN'S COLLEGE, May 1827.

1. Prove that

$$(a-b)^n = a^n \cdot \left\{1 - n \cdot \frac{b}{a-b} + \frac{n.(n+1)}{2} \cdot \frac{b^2}{(a-b)^2} - \frac{n.(n+1).(n+2)}{1.2.3} \cdot \frac{b^3}{(a-b)^3} + \&c. \right\}$$

and from this find the sum of the series

$$1 - \frac{1}{2} \cdot \frac{1}{2} + \frac{1.3}{2.4} \cdot \frac{1}{2^2} - \frac{1.3.5}{2.4.6} \cdot \frac{1}{2^3} - \&c. \text{ to infinity.}$$

2. Shew that $m.(x^{\frac{1}{m}} - 1) =$ Naperien logarithm x when m is infinite, and prove from this that Naperien logarithm x

$$= (x-1) \cdot \frac{2}{x^{\frac{1}{2}}+1} \cdot \frac{2}{x^{\frac{1}{4}}+1} \cdot \frac{2}{x^{\frac{1}{8}}+1}, \text{ \&c. to infinity.}$$

3. Investigate Cardan's rule for the solution of the cubic equation $x^3 - qx + r = 0$, and shew that when $\frac{r^2}{4} < \frac{q^3}{27}$, the expression for determining x may be reduced to the form $2\sqrt{\frac{q}{3}} \cdot \cos. \frac{\phi}{3}$, where ϕ is an angle whose cosine is $-\frac{3\sqrt{3}.r}{2q\sqrt{q}}$.

4. If A, B, C be the angles of a triangle, a, b, c the opposite sides, and θ the angle contained between b and a straight line drawn from A to the bisection of the opposite side, prove that

$$\theta = \frac{c}{b} \cdot \sin. A - \tfrac{1}{2} \cdot \frac{c^2}{b^2} \cdot \sin. 2A + \tfrac{1}{3} \cdot \frac{c^3}{b^3} \cdot \sin. 3A + \&c.$$

5. In a given rectangle inscribe a parallelogram of given area

and containing an angle equal to a given angle; and point out the limitations of the problem.

6. ACB is a circle, AB a diameter, draw any chord AC and join CB, and in AC produced take CP proportional to the triangle CAB: find the *locus* of P.

7. Two forces p and q acting at right angles on a material point have a resultant r; find two other forces of which the sum is the same but which acting at an angle θ have a resultant nr, and shew that this is always possible when $n > \frac{p+q}{\sqrt{(p^2+q^2)}}$.

8. A hemisphere is placed on a horizontal plane with its vertex downwards: find the height of a cone of the same base which placed upon the base of the hemisphere will make the equilibrium of the whole mass that of *indifference*.

9. An imperfectly elastic ball is projected from a point in the circumference of a given circle, and after three rebounds at the circumference returns again to the point of projection; find the direction of projection.

10. Find the moment of inertia of a rectangular parallelopiped about one of its edges, and also the time of a small oscillation.

11. A hollow cylinder of given radius but not of inconsiderable thickness *rolls* down a given inclined plane in a given time; find its thickness.

12. A right cone standing on its base is filled with mercury and water; divide it by a plane parallel to the base so that the pressure on the upper and lower curve surfaces shall be equal, the quantities of mercury and water being supposed equal in weight.

13. A cubical vessel of given size and weight is placed on a fluid of given specific gravity having a small orifice at the bottom; what time will elapse before the vessel is completely immersed?

14. In a vessel of fluid is placed a paraboloid with its vertex downwards; find its length so that the vertex of the paraboloid may just reach the bottom of the vessel.

15. Find the time of emptying the upper half of a paraboloid whose axis is parallel to the horizon through a small orifice in its vertex.

16. A glass meniscus of given radii and inconsiderable thickness floats on the surface of a bason of water of given depth; required the focus of refracted rays diverging from the bottom of the bason and passing nearly perpendicularly through the lens.

17. Find the nature and length of the caustic, when the reflecting curve is a cycloid and the radiating point in the centre of the generating circle.

18. Compare the quantity of light from the sun which falls on a given horizontal area on the longest day at two places, the latitudes of which are given.

19. Having given the latitude of the place, the day of the year, and the angle at which the ecliptic is inclined to the horizon; find the hour of the day.

20. Find the aberratic curve on the supposition that the Earth moves in an ellipse and is acted upon by a force perpendicular to the major axis.

21. Find at what point in an ellipse described about the focus the approach towards the centre of the ellipse is twice as great as that towards the focus.

22. A given body is projected with a given velocity in a horizontal direction along the inner surface of a horizontal cylinder of given radius from a given point not more than 90° from the lowest point; required its position after a given time.

23. Compare the resistance of the atmosphere against a spherical shot with that against a shot of the same weight formed into a hollow cylinder of given length and radius moving in the direction of its axis with the same velocity.

24. A chain of given length and thickness suspended by its extremities from two given points in the same horizontal line forms itself into a semicircle; find the law of the density.

QUEEN'S COLLEGE, MAY 1828.

1. PROVE analytically and geometrically that the perpendiculars drawn from the angles of a plane triangle upon the opposite sides intersect each other in one point.

2. Find a point in a spherical triangle equally distant from each of the angular points.

3. The area of a regular polygon of n sides inscribed in a circle is to the area of another regular polygon of $3\,n$ sides inscribed in the same circle as 1 to k; prove that $\cos.\theta = \frac{\sqrt{(3+k)}}{2\sqrt{k}}$, θ being the angle which a side of the last polygon subtends at the centre of the circle.

4. Find all the roots of the equation $x^7 - 1 = 0$ in a numerical form, and reduce each to the form of $\alpha + \beta\sqrt{-1}$.

5. Prove that $m \, . \, (x^{\frac{1}{m}} - 1) = \text{Nap. log.}x$ when m is infinite, and thence shew that

$$\text{Nap. log.}x = (x-1) \cdot \frac{2}{x^{\frac{1}{2}}+1} \cdot \frac{2}{x^{\frac{1}{4}}+1} \cdot \frac{2}{x^{\frac{1}{8}}+1} \; \&\text{c.}$$

6. Sum the following series:

$$5 + 9 + 51 + 231 \text{ to } n \text{ terms,}$$

$$\frac{1}{1.2.4} + \frac{3}{2.3.5} + \frac{5}{3.4.6} + \&\text{c. to } n \text{ terms and to infinity,}$$

$$\frac{1}{1.3} \cdot \frac{1}{3} - \frac{1}{3.5} \cdot \frac{1}{3^2} + \frac{1}{5.7} \cdot \frac{1}{3^3} + \&\text{c. to infinity.}$$

7. Solve the equation $u_{x+2} \, . \, u_x = e^x$, and find the general term of a series of which $S_x = m \, . \, u_x + nu_{x-1}$, S_x being the sum of x terms, and u_x being the xth term.

8. If two ellipses whose eccentricities are respectively e and $\frac{e}{2}$ be described on the same major axis, and θ and ϕ be the angles

which equal radii measured from the foci make with that axis; prove that

$$2\cos.\theta - \cos.\phi = \tfrac{3}{4}.e \text{ nearly.}$$

9. A given paraboloid whose axis coincides with the axis of x is cut by a plane whose equation is $z - qx = 0$, find the equations to the intersection of the two surfaces and determine its axes.

10. Investigate the equation to the curve in which the radius of curvature is always equal to the normal; the curve being supposed to be convex to the axis.

11. The directrix of a conical surface being a circle parallel to the plane of xy, find the equation to the surface; the vertex being supposed to be in the origin of the co-ordinates.

12. Find the ratio of the axes of a prolate hemispheroid such that when placed with its vertex on a horizontal plane, its equilibrium may be that of indifference.

13. A given beam of uniform thickness rests with one of its extremities in a given hemisphere, and the other against a vertical plane touching the hemisphere, find the position of equilibrium.

14. A body acted on by gravity slides down the curve of a vertical elliptic quadrant; determine its motion and the point where it leaves the curve.

15. Find the moment of inertia of a regular hexagon referred to an axis passing through one of its angular points and perpendicular to its plane, and find the time of one of its small oscillations.

16. A cylinder is filled with two fluids of different specific gravity which do not mix; find the pressure upon a section inclined at a given angle to the horizon, one half of which is in the upper fluid and the other in the lower.

17. If each particle of a fluid mass be attracted to a centre of force varying as the distance, and also by a given constant force acting in parallel lines; find the form which the mass will assume if a motion of rotation be given to it about an axis passing through the centre of force and parallel to the direction of the constant force.

18. Find the time of emptying a given cylinder inclined at a given angle to the horizon through a small orifice in its lowest point.

19. Determine analytically and geometrically in what point of a given straight line, another straight line given in magnitude and position will appear the greatest.

20. Having given the altitude of the Sun and of the violet colour in the primary rainbow, find the ratio of the sines of incidence and refraction.

21. Find the latitude of the place of observation from the following *data*, viz. the altitude and declination of the Sun, and the sum of the azimuth and hour-angle.

22. Explain the stereographic, orthographic and Mercator's projection of the sphere, and shew that the stereographic projection of every circle of the sphere is a circle, and find its diameter.

23. The particles composing the sides of a plane triangle attract with forces varying as the distance; find the orbit described by a body projected from any point within the triangle with a given velocity and in a given direction.

24. A body is placed in the centre of a hemispherical shell, the particles of which attract with forces varying as $\frac{1}{D^2}$; find the motion of the body.

25. Find the angle between the apsides when force $\propto \frac{x}{\sqrt{(1+x^2)}}$, x being the radius vector.

QUEEN'S COLLEGE, June 1829.

1. Shew that

$$\frac{1}{(a-b)(a-c)(x+a)} - \frac{1}{(a-b)(b-c)(x+b)}$$

$$+ \frac{1}{(a-c)(b-c)(x+c)} = \frac{1}{(x+a)(x+b)(x+c)}.$$

2. Through the point of intersection D of two straight lines drawn from two angles of a triangle to bisect the opposite sides, a

straight line $EDFG$ is made to pass, meeting two sides in E and F and the remaining side produced in G: prove that $\frac{1}{DE} - \frac{1}{DF} = \frac{1}{DG}$.

3. Find the sum of the present values of an infinite number of annuities of A£, to commence at the end of 1, 2, 3, &c. years respectively, and to continue for ever, at a given rate per cent.

4. If a, b, c, d; A, B, C, D be the sides and angles of any quadrilateral figure, taken in order, and $2S = a + b + c + d$; it is required to prove that

$$(\text{area})^2 = (S - a)(S - b)(S - c)(S - d) - abcd \cos.^2 \frac{A + C}{2}.$$

5. PSp, QCq are any two parallel straight lines drawn through the focus S and the centre C of an ellipse to meet the periphery: shew that $SP . Sp : CQ . Cq :: BC^2 : AC^2$.

6. Two bodies begin at the same time to descend from the top of two given inclined planes having a common altitude: prove that the line which joins them is always parallel to itself: find also its inclination to the horizon, and its velocity at any time.

7. Find the position of a luminous point in the same plane with the centres of three given spheres, so that the sum of their illuminated surfaces may be the greatest possible.

8. A cone with its vertex downwards is half filled with a fluid whose specific gravity is s: required the specific gravity of another fluid with which the vessel being filled up, the pressure of the mixture upon the whole surface will have a given ratio to its weight.

9. Let S_n, S_{2n}, &c., S_{pn} denote the sums of the nth, $(2n)$th, &c., (pn)th powers of the roots of the equation $x^n - L = 0$; then will

$$S_n + S_{2n} + \&\text{c.} + S_{pn} = nL\left(\frac{L^p - 1}{L - 1}\right):$$

equired a proof.

10. If the velocities of light in different media be unequal but uniform, prove that the time of its passage between two points situated in different media separated by a plane surface, will be the least possible when the cosines of the inclinations of its courses to the common surface are as the velocities.

11. Find the conic section to which the equation

$$(x+y)^2 - 2c(x-y) = 2c^2,$$

belongs, and determine the positions of its principal axis and focus.

12. Trace the curve whose equation is $y^3 = ax^2 - x^3$; determine its asymptote and the natures and positions of its singular points.

13. If the sum of the angles of a spherical triangle be equal to three right angles, it is required to prove that

$$\sin.2A = -\frac{\cos.\frac{1}{2}(b+c)\cos.\frac{1}{2}(b-c)}{(\sin.\frac{1}{2}b \sin.\frac{1}{2}c)^2}.$$

14. A body describes a cycloid by means of a force acting perpendicularly to its base: prove that corresponding to the arc θ of the generating circle, the force, velocity, and time from the vertex, are respectively as $\sec.^4\frac{\theta}{2}$, $\sec.\frac{\theta}{2}$, and $\theta + \sin.\theta$; and find their actual values when the intensity of the force at the highest point is represented by μ.

15. If m be the number of plane angles forming each solid angle, and n be the number of sides of each plane face, of a regular polyhedron; prove that $\frac{1}{m}+\frac{1}{n}$ must be greater than $\frac{1}{2}$, and thence determine the number of the Platonic bodies.

16. Integrate $\frac{x}{c^x}$, and by means of the integral find the sum of n terms of the series $\frac{2}{c}+\frac{3}{c^2}+\frac{4}{c^3}+$ &c.; also in infinitum, when possible: and shew that

$$(1^3-1)+(2^3-2)+(3^3-3)+\text{\&c. to } n \text{ terms} = \frac{n+2}{4}(n^3-n).$$

17. If L, l, K, k, be the true latitudes and longitudes of the Moon and a fixed star, and D their angular distance from each other, prove that

$$\sin.^2\frac{D}{2} = \sin.^2\frac{L-l}{2} + \cos.L\cos.l\sin.^2\frac{K-k}{2};$$

and adapt the formula to logarithmic computation.

18. Integrate

$$\frac{(x^2+1)x\,dx}{x^4+x^2+1},\quad \frac{dx}{x^m(a^m+x^m)^{\frac{1}{m}}},\ \text{and}\ \frac{x\,dx}{(a-x)^{\frac{3}{2}}(a+x)^{\frac{5}{2}}};$$

find also $\iint \frac{dx^2}{1-x^2}$ between the limits of $x=0$ and $x=1$; and determine the relation of x and y in the equation

$$(a+x)^{\frac{3}{2}}dy + (b-y)\sqrt{(a+x)}dx = (b-y)^{\frac{3}{2}}dx.$$

19. If a body describe an ellipse about the focus, find the effect produced upon the axis major by a small alteration in the intensity of the force at any given point, and the consequent motion of the apsides.

20. In a revolving fluid spheroid wherein gravity is everywhere perpendicular to the surface, prove that

the sensible gravity at the latitude $l = g\sqrt{\frac{1-e^2}{1-e^2\sin.^2 l}}$,

g being the polar gravity, and e the eccentricity of the spheroid.

21. If the Sun and Moon be in the equator, and the Moon in octants,

the angular distance of high water from the Moon $= \frac{1}{2}\cos.^{-1}\frac{M^2-S^2}{M^2+S^2}$,

and the whole magnitude of the tide produced $= \sqrt{(M^2+S^2)}$: required a proof.

22. Find the radii of greatest and least curvature at any point of an ellipsoid, and also the radius of curvature of a section through the axis of z making an angle θ with the plane of yz.

23. Apply the Calculus of Variations to determine the nature of the curve which joining two given points shall be described in the least time possible by a body whose velocity varies as its distance from a third point given in position.

24. The Earth being considered a spheroid of small eccentricity, prove that the effect of the Sun upon the matter exterior to the inscribed sphere, to turn the sphere about its centre, is the same as upon $\frac{1}{5}$th part of the protuberant matter collected at the equator.

QUEEN'S COLLEGE, MAY 1830.

1. EXTRACT the fifth root of the binomial $76 + 44\sqrt{3}$.

2. Reduce $3\sqrt[4]{-1} + 5\sqrt[8]{-1}$ to the form of $\alpha \pm \beta\sqrt{-1}$.

3. Determine the quantity which multiplied into $\sqrt[3]{3} + \sqrt[4]{4}$ will make it rational.

4. If in the expansion of $(1 + x)^m$, K and L be any two consecutive coefficients; prove that the coefficients after L will be

$$L \cdot \frac{Lm - K}{K.(m+2) + L}; \quad L \cdot \frac{Lm - K}{K.(m+2) + L} \cdot \frac{L.(m-1) - 2K}{K.(m+3) + 2L},$$
&c.

5. There is a series of numbers, each of which when divided by p, $2p$, $3p$, &c. in succession, gives remainders $p - 1$, $2p - 1$, $3p - 1$, &c. respectively; find the sum of n terms of this series, beginning with the least.

6. There are two light-houses, the position and the magnitude of whose lights are known; find the path which a vessel must describe so that the quantities of light it receives from each may always be in the same ratio.

7. A given sector of a circle is immersed in a fluid, with its axis vertical, and its vertex coincident with the surface: divide it by a horizontal line into two parts which shall sustain equal pressures.

8. Compare the times of emptying the upper half of a paraboloid whose axis is horizontal, through a small orifice in its vertex, with that of emptying the lower half through a small orifice in its lowest point.

9. A chain fastened by two tacks in the same horizontal line, forms itself into a common parabola; find the law of its thickness.

10. When one curve surface rests upon another, shew under what circumstances the equilibrium is stable, unstable, and indifferent; and when one paraboloid of revolution, cut off by a plane perpendicular to its axis, rests with its vertex upon that of another of equal parameter, the axis of both being vertical; find the length of the axis of the upper paraboloid so that the the equilibrium may be that of indifference.

11. A uniform beam is connected at one of its extremities by a hinge, to a horizontal rod, which is at liberty to move in a horizontal plane about a given point; find the velocity of revolution, so that the beam may rest inclined at a given angle to the rod.

12. A body acted on by gravity slides down the convex side of a vertical elliptic quadrant; find the point at which it will leave the curve, and also the point at which it will strike the horizontal plane.

13. A small ring slides down a straight rod, while the rod describes a conical surface, with a uniform velocity; find the path described by the ring and the time of moving down at a given length.

14. Find the density of the air at any point below the Earth's surface, the temperature being supposed the same, and the force of gravity varying as the distance; and explain the nature of the correction to be applied if the temperature vary.

15. Two stars are observed to rise at the same time in a given latitude, and when one has reached the prime vertical the other is observed to be on the six o'clock hour line; having given the right ascension and declination of the one, find the right ascension and declination of the other.

16. Find the equation to the orbit described, the angle between the apsides, and the points in which the orbit intersects itself, when a body is projected about a given centre of force, varying as $\frac{m}{D^2} + \frac{n}{D^3}$, with a velocity due to a finite space.

17. Apply the differential equations of motion to determine the time of vibration of a chord of given length (a) stretched by a given weight, the equation to the initial curve being

$$y = m \,.\, \sin.\frac{\pi}{a} \,.\, x;$$

and shew that it is the same as that determined from the general equation of vibrating chords.

18. Find generally the content of a solid polyhedron, and apply the expression to a dodecahedron.

19. Integrate $\frac{\sin.^2\phi d\phi}{1 + e . \sin.^2\phi}$; and shew the use of integration in determining the mean annual motion of the Moon's nodes.

20. In the expansion of e^{e^x} determine the coefficient of x^n.

21. Prove that

$$1 + \frac{n . \cos.\theta}{1} + \frac{n . (n-1)}{1 . 2} . \cos.2\theta + \&c. = 2^n \left(\cos.\frac{\theta}{2}\right)^n . \cos.\frac{n\theta}{2};$$

and that $\sin.\theta \sin.3\theta .. \&c. \sin.(2n+1) . \theta = \frac{1}{2^n}$

$$\text{if } (2n+1) . \theta = \frac{\pi}{2};$$

find also the sum of the series

$$1 + 2 \cos.\theta + 3 . \cos.2\theta + \&c. \text{ to } n \text{ terms.}$$

22. Integrate the following equations of differences:

$$(1). \ u_{x+2} - a^2 u_x = x^3.$$

$$(2). \ u_{x+1} - a^x u_x = 0.$$

23. Apply Lagrange's theorem to determine y from the equation

$$y = 1 + m . e^y.$$

24. Find the curve surface in which the distance of any point measured along the normal from the plane of x and z, is equal to m times the distance of the same point from the plane of y and z, measured along the same line.

CAIUS COLLEGE, Dec. 1828.

1. Find the time in which a circular basin, half a mile in diameter, and 10 feet deep, will be half emptied through a flood-gate 15 feet wide.

2. A candle is placed on a circular table; find the nature of its shadow on the floor.

3. Find the inclination of that diameter of a circle to the horizon, the time through which = 2 time down the vertical diameter.

4. Explain the principle on which achromatic telescopes are constructed.

5. A perfectly elastic ball is projected at a given angle, from a given point in an horizontal plane, and after rebounding, strikes a given object; find the velocity of projection.

6. Determine the length of that chord of a circle passing through the lowest point, down which a body acquires $\frac{1}{n}$th of the velocity in falling down the diameter.

7. The specific gravity of air being a, and that of water unity: if W be the weight of a body in air, and W_1 in water, prove that its weight in vacuo $= W + \frac{a}{1-a} \cdot (W - W_1)$.

8. Explain the method of measuring altitudes by means of the barometer and thermometer.

9. If two weights acting upon a wheel and axle without weight put the machine in motion, find the pressure on the axis.

10. Find the distance of the point of suspension from the centre of a given sphere, that the time of oscillation may be a minimum.

11. Find the image of the arc of a circle placed before a double convex lens, and not concentric with it.

12. Find the time of oscillation of a body in a cycloid, in a medium whose resistance is constant.

13. The radii and distance of the centres of two side wheels of a coach being given, a particle driven from the highest point of the hind wheel falls on the highest point of the fore wheel: what is the velocity of the coach?

14. The specific gravity of gold is 18000, of silver 11000, of air $1\frac{1}{5}$; the radius of the Earth is 4000 miles, the height of an homogeneous atmosphere $5\frac{1}{2}$ miles, and the whole weight of the atmosphere is equal to the weight of a sphere composed of a mixture of gold and silver whose diameter is 55 miles: required the proportion of gold to silver.

CAIUS COLLEGE, Dec. 1828.

1. If l be the length of a straight line, drawn from the vertex C of a triangle bisecting the base $(2c)$; shew that the area of the triangle $= \frac{l^2 - c^2}{2} \cdot \tan.C$.

2. Prove that

$$\sin.3^\circ = \tfrac{1}{8}.\{(1-\sqrt{3}).\sqrt{(5+\sqrt{5})} + (1+\sqrt{3}).\sqrt{(3-\sqrt{5})}\},$$

3. Draw through a given point in the diameter of a circle a chord which shall form with the lines joining its extremities with either extremity of the diameter, the greatest possible triangle.

4. Eliminate a and b from the equations $ax^2 + by^2 = ax + by$.

$$ax^2 + by^2 = bz.(y-x).$$

5. Solve the equation $(9+5\sqrt{3}).x^2 + (15+7\sqrt{3}).x+6=0$, and exhibit the roots under the form of binomial surds.

6. Sum $4^2 + 2^2 + 1^2 + \ldots\ldots\ldots\ldots$ to n terms.

$1.2 + 4.5 + 7.8 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$5.3 + 3.1 - 1.1 + 1.3 + 3.5 \ldots\ldots\ldots$

7. Find by the method of continued fractions, a series of fractions converging to $\sqrt{(17)}$.

8. Sum the series

$(\cos.a + \sqrt{-1}.\sin.a) + (\cos.a + \sqrt{-1}.\sin.a)^2 + \ldots\ldots$ to n terms,

and thence deduce the sums of

$\cos.a + \cos.2a + \cdots$ and $\sin.a + \sin.2a + \ldots\ldots$ to n terms.

9. A person having a capital of £a, spends annually a sum b, greater than the interest of his capital: in how many years will it be exhausted?

10. Find the value of $\sin.x$ in the equation

$$\sin.(x-a) = \sin.(x-2a).$$

11. Find whole positive values of x and y in the equation

$$6x - 5y = 1,$$

and find the sum of the first twenty values of y.

12. One thousand pounds is invested in the three per cent consols at 63 per cent, and sold out afterwards at 64: required the sum gained by the speculation.

13. If squares be described on the sides of any triangle and the angular points of the squares be joined; the sum of the squares of the sides of the hexagonal figure thus formed is equal to four times the sum of the squares of the sides of the triangle.

14. If all the angular points of a regular polygon of n sides be joined, and r be the radius of the circumscribing circle; the sum of all lines, including the periphery of the polygon, is equal to

$$nr \cot. \frac{\pi}{2n}.$$

15. If from a given point straight lines be drawn cutting an ellipse, the locus of the middle points of the chords is an ellipse similar to the given ellipse.

FOR SCHOLARSHIPS.

BELL'S SCHOLARSHIP, 1826.

1. Required the charge on £573. 18*s*. 6½*d*. at 12¾ per cent.

2. After deducting a charge of 8¾ per cent. on a certain sum, and then a charge of 6⅓ per cent. on the remainder, the result is £.256. 15*s*. Required the original sum.

3. Reduce 13*s*. 7¼⅛*d*. to the decimal of a pound: also $\frac{1}{13}$ of a shilling to the decimal of a crown.

4, Two clocks point to XII at the same instant; one of them gains 7″, and the other loses 6″ in 12 hours: after what interval will one have gained half an hour of the other? and what o'clock, exactly, will each then shew?

5. If two angles of a triangle be equal, shew that the sides subtending them are also equal.

6. Prove that, in a right-angled triangle, the square on the side subtending the right angle, is equal to the square on the sides containing the right angle.

7. If a straight line be divided into two equal, and also into two unequal parts, prove that the squares of the two unequal parts are, together, double of the square of half the line, and of the square of the line between the points of section.

8. If a straight line touch a circle, and, from the point of contact, a straight line be drawn cutting the circle, shew that the angles made by those lines will be equal to the angles in the alternate segments of the circle.

9. Demonstrate the proposition usually cited *Ex æquo.* [*Euclid,* Book 5, Prop. 22.]

10. Shew that the equiangular parallelograms have to one another the ratio compounded of the ratios of their sides.

11. Prove that two straight lines, which are each of them parallel to the same straight lines, and not in the same plane with it, are parallel to each other.

12. From a given point *without* a circle, draw a straight line which will cut off a quadrantal arc of the circle.

13. In a right-angled triangle, a and b are the sides containing the right angle, and c the side subtending it: prove that the radius of its inscribed circle $= \frac{1}{2}(a + b - c)$.

14. Multiply

$$a^{n-m} + a^{n-2m}b^m + a^{n-3m}b^{2m} \ldots + a^m b^{n-2m} + b^{n-m} \text{ by } a^m - b^m.$$

Also shew that $\dfrac{\sqrt{(20)} + \sqrt{(12)}}{\sqrt{5} - \sqrt{3}} = 8 + 2\sqrt{15}$.

15. Solve the following equations:

(1). $a + x - \sqrt{(a + x)} = a - x\sqrt{(a - x)}$;

(2). $\dfrac{8}{x^3} + 2 = \dfrac{17}{x^{\frac{3}{2}}}$;

(3). $\dfrac{x^3 - 10x + 1}{(x - 3)^2} = x - 3$;

(4). $\left.\begin{aligned} x + y &= a \\ x^5 + y^5 &= b^5 \end{aligned}\right\}$;

(5). $\frac{2}{3}x^2 + \frac{3}{4}x = \frac{4}{5}$; (shew that $x = {\cdot}6689$).

16. The difference between two numbers is 24, and their arithmetic mean exceeds their geometric mean by 6: required the numbers.

17. Prove that

$$(x + a)^m = x^m + max^{m-1} + \frac{m(m-1)}{2}a^2x^{m-2} + \&c.;$$

and shew that if the nth term of the series be multiplied by

$$\frac{m - (n - 1)}{n} \cdot \frac{a}{x},$$

the product will be the $(n + 1)$th term.

18. If $N = n \times 10^r$, shew in what respect the logarithms of N and n will differ; and having given the logarithm of 2·651 (= 0.4234079), state the logarithms of 2651, and of ·002651.

19. Find the sum of the successive mean proportionals between

$$a,\ ar;\ ar,\ ar^2;\ ar^2,\ ar^3;\ ar^3,\ ar^4;\ ar^4,\ ar^5;\ \cdots\ ar^{n-2},\ ar^{n-1};$$

Also sum the following progressions:

$$2\tfrac{2}{3} + 3\tfrac{1}{2} + 4\tfrac{1}{3} + \text{\&c. to 27 terms.}$$

$$\tfrac{5}{6} + \tfrac{5}{8} + \tfrac{15}{32} + \tfrac{45}{128} + \text{\&c. } \textit{in inf.}$$

20. Find an equation, each of whose roots is less by three, than the corresponding root of the equation $x^3 - 15x^2 + 71x - 105 = 0$; and from thence determine the roots of the given equation.

21. In an oblique-angled triangle, having given two sides a and b, and B the angle subtending b, state the logarithmic computation for finding A, the angle opposite to a. Explain also why, in some cases, A will have two values, in others only one.

22. Treatises of Trigonometry usually give two methods of finding the angles of a plane triangle, from the sides. Describe the two processes; and prove the leading proposition on which each method depends.

23. Prove that $\sin.\frac{1}{2}a = \sqrt{(\frac{1}{2}r^2 - \frac{1}{2}r\cos.a)}$,

and that $\cos.\frac{1}{2}a = \sqrt{(\frac{1}{2}r^2 + \frac{1}{2}r\cos.a)}$;

also from the first expression, find the sin.22°, 30′; and from the second, the cos.15°.

24. Shew that, in the equilibrium of two weights, acting perpendicularly to the arms of a straight lever, the weights are reciprocally proportional to their distances from the fulcrum.

25. Prove that, if a body be moved from rest by the action of an uniform force, the space described will vary as the square of the time.

26. P and Q are connected by a string passing over the top of an inclined plane: P, descending down the altitude, draws Q up the plane: given the length and inclination of the plane, find the time of Q's ascent to the top of the plane, and the velocity acquired.

27. A clock loses a second every hour; required the alteration to be made in the pendulum; the length of the true seconds' pendulum being 39·2 inches.

28. Given the velocity and direction of projection of a body, to find the horizontal range, the time of flight, and the greatest altitude.

29. Shew that, if circles be uniformly described by bodies acted upon by forces in the *centers*, those forces vary as $\frac{V^2}{R}$, and also as $\frac{R}{P^2}$.

30. Prove that, if a body describe a circle uniformly, the arc described in any time is a mean proportional between the diameter of the circle, and the space which the body would fall through in the same time, if acted upon by the force which retains the body in the circle.

TRINITY COLLEGE, 1820.

1. FIND the value of £.3·869, and 365·24215 days.
——— square root of 0·676, and 6·76.
——— amount of $\frac{2}{3}$, $\frac{1}{5}$, $\frac{9}{7}$, $25\frac{6}{11}$.

2. If twenty men, in digging a canal, must pump out six tons of water daily, in order to excavate 160 cubic yards in a week, how many cubic yards can thirty men excavate in a week, supposing them to be obliged to pump out eight tons of water daily?

3. If S denote the sum of the even terms, and S' the sum of the odd terms, of the expansion of $(a+b)^n$; then $S^2 \sim S'^2 = (a^2 \sim b^2)^n$.

4. Solve the equations, $\frac{6}{x+1} + \frac{2}{x} = 3 \cdots\cdots (A)$

$$x^3 - 12x + 6 = 0 \cdots\cdots (B)$$

$$x^3 - 2x^2 - 13x + 20 = 0 \cdots\cdots (C)$$

$$x^4 - 4x^3 + 5x^2 - 4x + 1 = 0 \cdots\cdots (D)$$

5. Find the roots of an equation of this form by construction, viz. $x^3 - 3r^2x - q = 0$.

6. Compute the numerical value of the side of a regular decagon inscribed in a circle, whose radius is ten inches.

7. The three angles of a plane triangle being given, and the distances of the three angular points from a given point within the triangles, to find the sides.

8. The complement of the hypothenuse of a right-angled spherical triangle cannot exceed the complement of either of the other sides.

9. If about the three angular points of a spherical triangle three great circles be described, the triangle formed by these latter will have its sides measures of the angles of the original triangle, or of the supplements of those angles.

10. In a given parabola, to draw a diameter which shall make with its ordinates an angle equal to a given rectilineal angle.

11. Two vertical straight lines being given, to place them at such a distance asunder in the same horizontal plane, that a heavy body shall be as long in falling down the greater, and then moving with its acquired velocity to the less, as it would be in falling down the less vertical, and moving with its acquired velocity to the greater.

12. When a radiant sphere shines upon an opake sphere, the breadth of the illuminating portion of the former has for its measure the same number of degrees as the dark portion of the latter.

13. The distance being given to which a fluid spouts from a given orifice in the side of a cylindrical vessel, to find by a geometrical construction the height of the fluid's surface in the vessel.

14. The right ascensions and declinations of three stars being given, and the times between their passages over the same vertical wire of a telescope, to find the latitude of the place; one of the stars being supposed in the equator.

15. The altitude of the Sun when due west, and also at six o'clock P.M. being given, find the latitude and Sun's declination.

16. Having the focus of incident rays upon a medium terminated by two plane sides inclined at a given angle, find the focus of emergent rays.

17. The exact quantity of the year being 365·24215 days, explain the reason of the corrections in the civil year introduced by Julius Cæsar and Pope Gregory.

18. Give the theory of the Trade Winds.

19. Prove that part of the equation of time which arises from the obliquity of the ecliptic to be a maximum when the longitude of the Sun equals the complement of its right ascension.

20. Compare the surface of a sphere with the area of its great circle, and its magnitude with that of its circumscribing cylinder.

TRINITY COLLEGE, 1821.

1. To find the locus of the extremities of all the straight lines that can be drawn from the circumference of a given circle, toward the same parts, each of them equal and parallel to a given finite straight line.

2. To find the centre of a given ellipse.

3. To construct the curve of which the equation is

$$ax^2 + ay^2 + bx + cy + d = 0.$$

4. If the product of any two given numbers be a square, each of the two given numbers is the product of two factors, such that the four factors are proportionals.

5. Solve the following equations:

(1). $\dfrac{12 + 2x}{x + 3} + \dfrac{4x - 3}{2x + 1} - \dfrac{4x - 1}{x - 1} = 0.$

(2). $a^{n - \frac{r}{x}} - b^{x-p} \times c^{qx} = 0.$

(3). $2xy + x + y - 195 = 0$, (to find the integral values of x and y).

(4). $\left\{\begin{array}{l} y^2 - x^2 - 90000 = 0 \\ yx - 300y - 125x = 0 \end{array}\right\}.$

6. If none of the coefficients of the equation

$$x^n + ax^{n-1} + bx^{n-2} + \&c. + g = 0$$

be fractional, it cannot have a fractional root.

7. To compare the chance of throwing 7, with the chance of throwing 8, at one throw, with three common dice.

8. Sum the following series.

(1). $1 + 3 + 27 + 64 +$ &c. to n terms.

(2). $\frac{1}{3} + \frac{4}{9} + \frac{9}{27} + \frac{16}{81} +$ &c. *ad infinitum.*

(3). $\frac{5}{1.2.3.4} + \frac{7}{2.3.4.5} + \frac{9}{3.4.5.6} +$ &c. to n terms.

9. (1). Find the fluent of $\frac{x\dot{x}}{\sqrt{\{(x^2 - a^2).(b^2 - x^3)\}}}$,

of $\frac{\dot{z}}{x^2\sqrt{(a^2 - v^2)}}$,

z being a circular arch, of which x is the cosine, and v the tangent;

and of $\frac{x^{\frac{1}{3}}\dot{x}}{a + bx^3}$.

(2). Solve the fluxional equation $x\dot{y}^2 - y\dot{x}\dot{y} - \frac{y^2}{a}\dot{x} = 0$.

10. If $A + B$ be less than a semicircle,

$$\sin.\frac{A \sim B}{2} : \frac{\text{ver. sin.}A \sim \text{ver. sin.}B}{2} :: \text{rad.} : \sin.\frac{A + B}{2}.$$

11. If P be put for the semi-perimeter of a spherical triangle, the sides of which are denoted by a, b, c, and the opposite angles by A, B, C,

$$\cos.^2 \tfrac{1}{2}A = \frac{\sin.P \sin.(P - a)}{\sin.b \sin.c}.$$

12. The upper extremity of an inclined plane being given, to determine its position, so that the time shall be a minimum, in which a body falls down it, and afterwards moves to a given point in the horizontal plane, with that part of its acquired velocity, which is not destroyed by its impact on the horizontal plane.

13. A given sphere, and its circumscribing cylinder, of the same uniform density, being supposed to revolve round their axes, with equal angular velocities, to compare their momenta.

14. A hollow sphere is to be formed of a substance, the specific gravity of which is greater than that of air, in the ratio of n to 1, and is afterwards to be filled with gas, the specific gravity of which is less than that of air, in the ratio of 1 to m; the thickness of the shell being given, to find its diameter so that it may float in the air.

15. To describe the construction, and determine the magnifying power of a Compound Microscope.

16. To describe the construction of an Achromatic Lens, and explain the reasons of that construction.

17. To determine the Sun's parallax, from observations made on the transit of Venus.

18. The times of a star's transit over the meridian, and over two vertical circles at given distances frem the meridian, having been observed, to compute the latitude of the place of observation, in terms of the azimuths, and hour-angles thus given.

19. To determine under what circumstances of the velocity of projection, a body, projected from a given point, in a given direction, and acted upon by a force inversely proportional to the mth power of the distance from the centre, will come to the centre, or to an apse.

20. If a body, acted upon by the constant force of gravity, fall down the concave side of a circular arch, the tangent of which, where the body begins to fall, is perpendicular to the horizon, to find the point where the pressure on the curve shall be equal to an nth part of the weight of the body.

TRINITY COLLEGE, 1827.

1. PARALLELLOGRAMS upon the same base, and between the same parallels, are equal to one another.

2. If a straight line be divided in extreme and mean rate, the parts into which it is divided are incommensurable with each other.

3. If 3lbs of tea be worth 4lbs of coffee, and 6lbs of coffee be worth 20lbs of sugar, how many pounds of sugar may be had for 9lbs of tea?

4. Solve the equation $\frac{x+2}{x-2} + \frac{x-2}{x+2} = 5 + \frac{1}{x+2}$.

5. Supposing $\frac{a}{\sqrt{(a^2+b^2)}} = \cos.\theta$, it is required to shew that

$$\sqrt[3]{(a+b\sqrt{-1})} + \sqrt[3]{(a-b\sqrt{-1})} = 2\sqrt[6]{(a^2+b^2)}\cos.\frac{\theta}{3}.$$

6. Prove that $\sin.(A+B) = \sin.A\,.\cos.B + \cos.A.\sin.B$.

7. Given the Napierian logarithm of $4 = 1{\cdot}3863$, it is required to assign the Napierian logarithm of 5 to 4 places of decimals.

8. Find the locus of the vertex of a △, where one of the angles at the base is always double of the other.

9. Investigate the differential equation of the tangent of a curve.

10. Integrate the following differentials:

$$(\alpha).\ \frac{dx}{1-x+x^2},$$

$$(\beta).\ \frac{dx}{\cos.x},$$

$$(\gamma).\ \frac{2ydx}{(x+y)^2} - \frac{2xdy}{(x+y)^2}.$$

11. The sum of the squares of any two semi-conjugate diameters of an ellipse is equal to the sum of the squares of the two semi-axes.

12. The area of a spherical △ is proportional to the excess of the sum of its angles above two right angles.

13. In any lever, the two forces required to keep it at rest, are inversely as the perpendiculars from the fulcrum upon their directions, "The principles and propositions, whether assumed or proved, which are made use of in demonstrating this proposition, are required to be distinctly stated."

14. A cylinder of given height and radius, whose lower half is iron and upper half wood, is placed with its base upon an inclined plane, what must be the proportion between the friction and pressure as well as the inclination of the plane, so that it may just be prevented from sliding and falling over?

15. If s be the space, t the time, and f the force, then

$$f = \frac{d^2s}{dt^2}.$$

16. The force which accelerates the descent of a body in a cycloid, is proportional to the distance from the lowest points; prove this, and shew that the time of oscillation is the same both in great and small arcs.

17. Find the law of force, by which a body may describe a circle, the force tending to an eccentric point; determine likewise, from the requisite data, the periodic time.

18. If a body be projected from any point with a given velocity and attracted to a centre of force, its velocity at any other given distance from the centre is independent of the direction of projection.

19. In ellipses of small eccentricity, the greatest equation of the centre is nearly equal to twice the eccentricity.

20. An inflexible and imponderable rod, to which two equal weights are attached, is fixed to a given point, round which it vibrates; find the time of its making a small oscillation.

21. Find the focal length of a double concave lens.

22. In the modern construction of the Newtonian telescope, the small mirror is replaced by a prism; explain the principle and advantages of this substitution.

23. The water in a clepsydra, whose form is a solid of revolution round a vertical axis, descends through equal spaces in equal times; find the equation of the generating curve.

24. Is the Sun's place affected by aberration?

25. An iron globe descends in water; find expressions for the velocity acquired, and the space described in a given time from the beginning of the motion.

26. If the equations of two planes be

$$Ax + By + Cr = 1,$$

$$ax + by + cr = 1;$$

then the cosine of the angle between them is

$$\frac{Aa + Bb + Cc}{\sqrt{(A^2 + B^2 + C^2)} \cdot \sqrt{(a^2 + b^2 + c^2)}}.$$

TRINITY SCHOLARSHIPS, April 1828.

1. Equiangular ▱s are to each other in the ratio compounded of the ratios of the sides. [*Euclid* 23, Book 6.]

2. Construct a regular tetrahedron, and compute the angle formed by two adjacent surfaces.

3. What is the discount of £70. 5*s*. 6*d*. due 7 months hence interest being allowed at 4½ per cent.?

$$\left(\text{Discount} = \frac{nrP}{1+nr} = \frac{\frac{7}{12} \times \frac{4\frac{1}{2}}{100} \times (70 + \frac{1}{4} + \frac{1}{40})}{1 + \frac{7}{12} \times \frac{4\frac{1}{2}}{100}}\right.$$

$$= \frac{63 \times (70 + \frac{11}{40})}{2463} = \frac{21}{40} \times \frac{2811}{821}$$

$$\left.= £.1\ 15s.\ 11\tfrac{1}{4}d.\ \tfrac{510}{821}\right).$$

4. Assign the values of x in the equation

$$\sin.(x + a) = \sin.(x - a).$$

(Expanding the functions of binomial angles, we get

$$2\cos.x \,.\, \sin.a = 0;$$

$$\therefore \cos.x = 0;$$

$$\left.\therefore x = \frac{\pi}{2},\ 3\cdot\frac{\pi}{2},\ 5\cdot\frac{\pi}{2},\ 7\frac{\pi}{2},\ \&c.\right)$$

5. Find integral and positive values of x in the equation

$$7x + 17y = 172.$$

6. Every equation has as many changes of sign from + to −, or from − to +, as it has positive and possible roots, and as many continuations of sign, from + to +, or from − to −, as it has negative and possible roots.

7. Explain the mode of transforming the equation of a curve from one system of co-ordinates to another, and apply the formulæ to

transform the equation $y^2 = 2ax + x^2$ to a system of rectangular co-ordinates, making angles of 45° with the former, and the co-ordinates of whose origin are 0 and $-a$ respectively.

8. Investigate the differential expression for finding the length of a curve, and apply it to find the length of the cycloid.

9. Integrate $\frac{dx}{1 + x + x^2}$, $\frac{z^3dx}{\sqrt{(2ax - x^2)}}$, and $\frac{dx}{\cos . x}$;

and find the sum of

$$\frac{1}{1 . 3} + \frac{1}{5 . 7} + \frac{1}{9 . 11} + \text{\&c.}$$

10. Find the equation of the curved surface traced out by one of the angles of a square, in constant contact with a spherical surface, without sliding upon its surface.

11. Find the centre of gravity of two or more bodies considered as material points.

12. A perfectly elastic ball let fall from the top of a tower meets another at half the height, of twice its weight, and which was projected upwards from the bottom at the same time; what are their motions after collision, and when will they reach the ground?

13. What is meant by the centre of percussion, and how is it found?

14. A double convex lens is placed at a given distance from a screen: to determine the position of the object, so that the image may fall upon the screen; is this always possible?

15. State the order of the colors in the primary and secondary rainbows, and the reason of their being the inverse of each other.

16. Determine the greatest height to which water can be raised by the common pump.

17. Investigate the precession of a star in right ascension.

18. The equation of time is Zero four times in the year, and at very unequal intervals of time.

19. Find the law of force to the focus of the logarithmic spiral, and find an expression for the whole time of descent, from a given distance, to the centre of force.

20. A body is projected from a given point in a given direction with a given velocity, and is repelled from a given point by a force varying as the distance: determine the orbit described.

21. An iron ball descends in a fluid: what is the limit of its velocity?

22. The Moon is retained in her orbit by the force of gravity.

23. Give the moment of inertion round any axis passing through the centre of gravity of a body, to find the moment round any other axis parallel to the former.

24. If d and d' be the lengths of a degree of the meridian at the equator, and at latitude λ respectively, and a and b the equatorial and polar diameters, then

$$\frac{a}{b} = \frac{\sin.\lambda}{\sqrt{\left\{\left(\frac{d}{d'}\right)^{\frac{3}{2}} - \cos.^2\lambda\right\}}}.$$

25. To find the nature of the curve when the normal bears a constant ratio to the part of the axis intercepted between the origin and the normal.

TRINITY COLLEGE, 1829.

1. EQUIANGULAR ▱s have to one another the ratio which is compounded of the ratios of their sides.

2. If two straight lines be ⊥ to the same plane, they are || to one another.

3. Required the value of the *Metre* of France in terms of the foot of Cremona; if 48 feet of Cremona = 56 English feet and the metre = 39·371 English inches.

4. Solve the equations

$$(\alpha).\quad \frac{x}{7-x} + \frac{7-x}{x} = 2\tfrac{9}{10}.$$

$$(\beta).\quad 1 - \sin.a = 2\sin.^2\left(x - \frac{a}{2}\right).$$

(γ). $ax^4 + 4x^3 - 6ax^2 + 4x + a = 0.$

(δ). $11x + 5y = 100.$

5. Prove that

$$\sin.A + \sin.B = 2\sin.\frac{A+B}{2}\cos.\frac{A-B}{2};$$

and also that $e^{\theta\sqrt{-1}} = \cos.\theta + \sqrt{-1}\,.\,\sin.\theta.$

6. Small spherical $\triangle^s$ may be considered rectilinear, without sensible error.

7. The roots of the reducing cubic equation in Des Cartes' solution of a biquadratic are quadruple of the roots of the reducing cubic equation in Euclid's Solution.

8. Determine the position of the centre and axes of the ellipse represented by the equation

$$Ay^2 + Cx^2 + Dy + Ex + F = 0,$$

all the coefficients being supposed positive.

9. Investigate the differential expression for the radius of the ⊙ of curvature.

What are the analytical and what the geometrical properties of the ⊙ of curvature, and how are they dependent upon each other.

10. Integrate

$$\frac{dx}{x\sqrt{(1+x^2)}},\quad \frac{dx}{(1+x^2)^3}\text{ (from } x = 0 \text{ to } 1\text{) and }\frac{dx\sqrt{(1-e^2x^2)}}{\sqrt{(1-x^2)}}$$

in a series which converges rapidly when e is nearly $= 1$.

11. Find the equation of the epicycloid when the radius of the generating ⊙ is equal to the radius of the base; and determine its area.

12. In the game of whist what is the chance of the dealer having the four aces?

13. In the lever the velocity of the weight is to the velocity of the power as the power is to the weight.

14. Prove that $v = \frac{ds}{dt}$; and $f = \frac{dv}{dt}$.

15. Find the length of a pendulum which would oscillate once in the period of a satellite at the Earth's equator.

16. A sphere rolls down an inclined plane; find the force accelerating its descent.

17. When two or more bodies move uniformly in straight lines their centre of gravity also moves uniformly in a straight line.

18. The power of a compound lens is the sum of the powers of the component lenses.

19. What is meant by the error of collimation in a telescope? How is it corrected?

20. Find the centre of pressure of a sluice-gate in the form of an isosceles △, whose base coincides with the surface of the fluid.

21. Find the precession of a star in right ascension.

22. What is the greatest number of eclipses of the Sun and Moon which can happen in a year?

23. To find the law of force to the focus of a logarithmic spiral.

24. Let the force $\propto \frac{1}{(\text{dist.})^3}$, and let a body be projected in any direction from a given point, with a velocity less than that in a ⊙ at the same distance, to determine the orbit described.

25. The length of a degree ⊥ to the meridian is always greater than that of the degree of the corresponding meridian.

26. At a given place, at a given hour, and on a given day, required the point of the compass on which a rainbow would appear.

TRINITY COLLEGE, APRIL 1830.

1. TRIANGLES upon equal bases and between the same parallels, are equal to one another.

2. Equal parallelograms, which have one angle of one equal to one angle of the other, have their sides about the equal angles reciprocally proportional and conversely.

3. Find the radius of a sphere described about a given regular tetrahedron.

4. Multiply 54·6 by 8·93 in the duodecimal scale: and if the units in the factors be feet, express the product in square yards, feet, and inches.

5. The sides of a triangle are 17·09 and 93·451, and the angle opposite to the greater of them is 93°.16′: find the angle opposite to the less.

$$\log.1709 = 3·2327421,$$

$$\log.93451 = 4·9705840,$$

$$\log.\sin.86°.44' = 9·9992938,$$

$$\log.\sin.10°.31' = 9·2614519,$$

6. Explain the theory and use of the columns of proportional parts, in tables of logarithms.

7. Solve the equations:

$$(\alpha).\ \frac{ax}{a-b} + \frac{bx}{b-c} = \frac{d}{a-c}.$$

$$(\beta).\ \frac{x}{x+2} - \frac{x-9}{3x-20} = \frac{9}{13}.$$

$$(\gamma).\ \sin.(x+a) + \cos.(x+a) = \sin.(x-a) + \cos.(x-a)$$

8. Eliminate x from the two equations

$$\left.\begin{array}{l} x^4 - (2y^2+1)\,x^2 + y^4 + y^2 - 20 = 0 \\ x^4 - 2yx^2 + y^2 - 49 = 0. \end{array}\right\}$$

9. In a game of pure skill, if *A*'s skill be to *B*'s as 3 to 1, what is the chance of *A*'s winning 3 games out of 4?

10. Define a differential, and from your definition find the differential of a^x.

11. Draw a tangent to a cycloid and determine its area.

12. All parallelograms which circumscribe an ellipse are equal to each other.

13. In what respect are the curve surfaces represented by the equations

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 - \left(\frac{z}{c}\right)^2 = 1,$$

and $$\left(\frac{x}{a}\right)^2 - \left(\frac{y}{\ }\right)^2 \ \left(\frac{z}{c}\right)^2 = 1,$$

distinguished from each other? Find the length of a normal to the former, intercepted between the surface and any one of its principal sections.

14. Integrate $\dfrac{x^2dx}{(x+a)(x+b)}$, $\dfrac{dx}{\sqrt{(ax+bx^2)}}$,

$\dfrac{d\theta \cos.\theta}{a+b\cos.\theta}$, and the equation

$$(a+y)\frac{dx}{dy}=x+y-\frac{xdy}{dx}.$$

15. Find the distance of the centre of gravity of any number of given bodies from a given plane; the distance of the centre of gravity of each body from the plane being given.

16. If P (5) draws up Q (3) by means of a string passing over a fixed pulley, find the force accelerating P's descent and the space described in t'' (10): the weight of the string and the inertia of the pulley being neglected.

17. Investigate the conditions of the equilibrium upon the screw.

18. The centres of oscillation and suspension are reciprocal.

19. Upon what do the magnifying power, brightness, distinctness, and achromatism of the astronomical telescope depend?

20. Find the deviation of a ray passing through a prism, whose refracting angle is considerable.

21. Show how to find the approximate solution of the equation, which expresses the relation between the eccentric and the mean anomaly.

22. Find the precession of a star in R. A.: when is it zero?

23. Investigate a formula to express the acceleration of the time of rising of a given star by the horizontal refraction.

24. Prove that the pressure upon any portion of a vessel filled with fluid of uniform density, is equal to the weight of a column of fluid whose base is the area of the surface pressed, and altitude the perpendicular depth of its centre of gravity, below the surface of the fluid.

25. Find the law of force acting in parallel lines, by which a body may be made to describe a portion of a circle.

26. A body is projected from a given point, in a given direction, with a given velocity above a given plane: find the orbit described when it is repelled from the plane by a force varying as its distance from it.

27. Determine the attraction of an oblate spheroid upon a particle situated at its equator.

28. The Moon is retained in her orbit by the force of gravity.

FOR FELLOWSHIPS.

TRINITY COLLEGE, 1819.

1. A GROCER had 150lbs. of tea, of which he sold 50lbs. at 9s. per pound, and found that he was then gaining only 7½ per cent. But he wished to gain 10 per cent. on the whole. At what rate must the remaining 100lbs. be sold that he may attain his wishes?

2. Solve the following equations:

$$(1).\quad \left.\begin{aligned} x + y + \frac{y^2}{x} &= 20 \\ x^2 + y^2 + \frac{y^4}{x^2} &= 140. \end{aligned}\right\}$$

$$(2).\quad \left.\begin{aligned} x - \sqrt{x} &= 3 - y \\ x - \sqrt{y} &= 4 - x. \end{aligned}\right\}$$

$$(3).\quad x^4 - 5x^3 + 7x^2 - 5x - 6 = 0.$$

(4). $x^x = a$ (by approximation).

3. In what time will a sum of money placed at compound interest double itself at 4¾ per cent.?

4. Required the value of the infinite series

$$\sqrt[4]{(18 + 189\sqrt[4]{(18 + 189\sqrt[4]{(18 + \&c.)})})}$$

5. Required the value of y in the following equations, when $x = 1$:

(1). $y = (1 - x) . \tan.\frac{\pi x}{2}$ (π = semi-circumference of a circle, rad. $= 1$).

(2). $y = \dfrac{x^x - x}{1 - x + \log.x}$;

6. Transform the equation $x^3 - 5x^3 + 8x - 5 = 0$ into one whose roots are the squares of the differences of every pair of roots: and show the mode of determining, from the transformed equation, the impossible roots in the original equation.

7. From a bag containing four white and eight black balls, three persons (*A*, *B*, and *C*) take each a ball in turn, viz. *A* first, then *B*, then *C*, and so on in succession, until the person, who first draws a white ball, wins. What are their respective chances?

8. In a spherical triangle, the two sides and the angle between them being given, find the base.

9. The vertical angle of an isosceles spherical triangle is always greater than the angle included between the chords of the equal sides.

10. Sum the following series:

(1). $2 + 6 + 12 + 20$ to n terms.

(2). $\frac{1}{1\,.\,2} + \frac{1}{4\,.\,5} + \frac{1}{7\,.\,8} + \frac{1}{10\,.\,11}$, &c. *ad infin.*

(3). $\cos.A + \frac{1}{2}\cos.2A + \frac{1}{3}\cos.3A +$ &c. to n terms.

11. Sum the series $\sin.\phi \times \sin.\theta + \sin.2\phi \times \sin.2\theta$ &c. to n terms; and show that when n is infinite, the sum $= 0$ whatever be the ratio of ϕ to θ, except that of equality.

12. A body attracted towards a centre by a force varying inversely as the square of the distance from the centre, meets at a given point of its rectilinear descent with a plane inclined at an angle of 45°. Required the time from the beginning of motion to its reaching the centre.

13. A cylindrical wheel, whose weight is P, unwinds itself from a string passing round its circumference, what weight (W) attached to the extremity of the string will be kept at rest on a plane of given elevation as P descends vertically?

14. A sphere and its circumscribing cylinder revolve round their common axis. Required the ratio of the *momenta* generated in a given time.

15. An homogeneous circular wheel vibrates edgeways, being suspended from a point in the circumference. Required its centre of oscillation.

16. Find the centre of gravity of the area included by the arc of a cycloid, by a tangent at the vertex, and by two rectangular ordinates equi-distant from the vertex.

17. In the common parabola the radius of curvature is equal to the cube of the normal divided by the square of the semi-parameter.

18. Trace the curve whose equation is $y = \pm \dfrac{ax - x^2}{\sqrt{(2ax - x^2)}}$, and find the angles at which it cuts the line of the abscissæ.

19. *O* is the centre of the circular arc *AB*. *OBT* is the secant. The exterior part *BT* is continually bisected in *P*. Required the area traced out by *OP*.

20. Two balls, *A* and *B*, are previous to motion at a given distance from each other in the same vertical line: from what height above the horizontal plane must *A* be let fall—so that *B*, which is perfectly elastic, may, after reflection, meet *A* at a given distance above the plane?

21. Two balls lying on a horizontal plane are connected by a string of unlimited length which passes through a ring in the plane. One of the balls is projected in a given direction with a given velocity and draws the other towards the ring. Required the curve which the projected ball describes.

22. Find the following fluents:

$$\int \dot{z} \,.\, \cos.z \,.\, \cos.nz,\ \int \dot{z} \,.\, \cos.^3 z,\ \int \frac{\dot{x}}{\sqrt{(1 - \varepsilon^{-x})}},$$

construct the fluent $\displaystyle\int \frac{\dot{z}}{z + 2az^2 + z^3}$, when a is less than 1;

and the relation of x to y in the equation

$$\dot{x}y - y\dot{x} = \dot{x}\sqrt{(x^2 - y^2)}.$$

23. In any curve referred to an axis, the ordinate is a maximum or a minimum, when in the equation $y = fx$, an odd number of differential coefficients becoming $= 0$, the differential coefficient of the succeeding order is negative or positive. And there is a point of contrary flexure when an even number of differential coefficients becoming $= 0$, the differential coefficient of the succeeding order is real and finite.

24. A body urged by a constant force in an uniform resisting medium is projected in a direction contrary to the action of the force with a certain velocity; it is required to determine the velocity at any point of the ascent, the resistance being supposed proportional to the square of the velocity. Find also the greatest height to which the body will ascend.

TRINITY COLLEGE, 1820.

1. In a plane triangle the vertical angle, the perpendicular, and the rectangle under the segments of the base being given, it is required to construct the triangle.

2. Solve the equation $x^3 - \frac{4x^2}{3} - \frac{17x}{3} + 2 = 0$, two roots being of the form a and $\frac{1}{a}$. And find the number of all the possible values in integer numbers of x, y and z in the equation

$$5x + 7y + 11z = 224.$$

3. What are the dimensions of the strongest rectangular beam, that can be made out of a given cylinder, when placed to the most advantage? and what is its strength, compared with that of the greatest square beam cut out of the same cylinder?

4. In the wheel and axle (the inertia of which may be neglected) required the ratio between the radii, when a weight (Φ) acting at the circumference of the wheel generates in a given time the greatest momentum in a weight (W) attached to the circumference of the axle.

5. Tangent of half the spherical excess

$$= \frac{\tan.\tfrac{1}{2}b \times \tan.\tfrac{1}{2}c \times \sin.A}{1 + \tan.\tfrac{1}{2}b \times \tan.\tfrac{1}{2}c \times \cos.A},$$

where b, c, and A are the two sides and included angles of a spherical triangle.

6. The excess of the Sun's longitude above its right ascension may be found by the equation

$$\tan.(L - \text{R. A.}) = \frac{\tan.^2\tfrac{1}{2}\Upsilon \times \sin.2L}{1 + \tan.^2\tfrac{1}{2}\Upsilon \times \cos.2L}.$$

7. Find an expression from which the effect of parallax upon the horary angle may be accurately calculated; the horizontal parallax, the polar distance of the heavenly body, and the time before or after transit, being the only given elements.

8. If an orifice were opened half way to the centre of the earth, what would be the altitude of the mercury in a barometer at the bottom of it, when the altitude at the surface is 30 inches.

9. A vessel formed by the revolution of a parabola round its axis is placed with its vertex downwards, in which there is an orifice one inch in diameter. A stream of water runs into the vessel through a pipe of two inches diameter at the uniform rate of eight feet per second. What will be the greatest quantity of water in the vessel, supposing the latus rectum to be six feet?

10. Find the present worth of the reversion of a freehold estate after the death of a person now sixty years of age, the rate of interest being given?

11. When a ray of light passes out of one medium into another, as the angle of incidence increases, the angle of deviation also increases.

12. To find the least velocity, with which a body projected at a given angle of elevation will not return to the earth's surface.—To find also the latus rectum of the orbit described, and the position of the axis.

13. Supposing the Moon's orbit at present to be circular, what would be the eccentricity of it and the periodic time, if the attraction of the earth were diminished $\frac{1}{n}$ part?

14. Find the sum of the following series:

(1). $\sin.(A) + \sin.(A + B) + \sin.(A + 2B)$ &c. ad infin.

(2). $\cos.A + \cos.3A + \cos.5A +$ &c. ad infin.

(3). $\frac{1^2}{1.2.3.4} + \frac{3^2}{2.3.4.5} + \frac{5^2}{3.4.5.6}$, &c. to n terms, by the method of increments.

15. Find the following fluents:

(1). $\int \frac{\dot{x}}{x^3(a + bx^2)}$, (2). $\int \frac{x\dot{x}}{\sqrt{(a + bx + cx^2)}}$.

(3.) $\int \sqrt{(a^2 - x^2)} \times x\dot{x}$, between the values $x = 0$, and $x = a$; and solve the fluctional equation $\frac{(\dot{x}^2 + \dot{y}^2)^{\frac{3}{2}}}{\dot{x}\ddot{y}} = a$.

16. *A* and *B* put down equal stakes,—*A* has *m* chances of success, and *B*, *n* chances. There are, moreover, *p* chances, which entitle both parties to withdraw their stakes,—to find the gain of *A*.

17. Two equal weights are placed at a given distance from each other on a straight rod supposed to be without weight. Find the point of suspension, so that the pendulum may vibrate seconds.

18. Construct the curve whose equation is

$$(a - x)^2 . (a + x) = x^2y^2,$$

and shew whether there are any cusps.

19. Invenire incrementum horarium areæ, quam luna, radio ad terram ducto, in orbe circulari describit.

20. Let the force to a centre vary as the distance, to find all the various curves, along which a body may move, so that its oscillations may be isochronous.

21. Given the diameters of two planets, and the periodic times and distances of their respective satellites; compare their densities and quantities of matter.

TRINITY COLLEGE, 1827.

1. STATE the difference between Geometrical Analysis and Synthesis, and give an example. Why are ordinary operations in algebra termed *analytical*?

2. If a, b, c be the angles which a $\perp^r$ upon a plane, drawn from

the co-ordinates, makes with the three rectangular axes, and if x, y, z be the co-ordinates of any point in the plane, then

$$x \cos.a + y \cos.b + z \cos.c = \delta.$$

3. Every equation of an odd number of dimensions has at least one possible root.

4. What are the analytical and geometrical properties of the circle of curvature, and in what manner are they deducible from each other?

5. If $y = F(z + xu)$ where u is a function of y, then

$$\frac{dy}{dx} = u \cdot \frac{dy}{dz}.$$

6. Determine the locus of a $\perp$ upon the tangent of a curve whose equation is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

drawn from a point whose co-ordinates are $\sqrt{(a^2 - b^2)}$ and 0.

7. If D be the observed angle between any two equals whose elevations or their sines are a and a' respectively, and if H be the corresponding angle reduced to the horizon, then

$$H - D = \frac{1}{4 \sin.1''} \cdot \left\{ (a + a')^2 . \tan.\frac{D}{2} - (a - a')^2 . \cot.\frac{D}{2} \right\}.$$

8. A cylinder pierces a sphere, its axis not passing through the centre: find, from the requisite data, the volume which is cut out.

9. The series

$$\sin.A + \sin.(A + B) + \sin.(A + 2B) + \&c.$$

is a recurring series: find its scale of relation, and its sum to n terms.

10. The moment of the resultant of two forces is the algebraical sum of the moments of the component forces.

11. Explain fully the method of determining the accelerating force of P's descent, when P raises W over a single moveable pulley: the inertia of the pullies being taken into account.

12. A given conical vessel filled with water, is placed with its base on a horizontal surface: compare the pressures on its base and sides with the weight of the fluid.

13. The conjugate foci in a spherical reflector move in the same direction upon its axis and coincide at the surface and centre.

14. What is the Newtonian hypothesis to explain the reflection and refraction of light? Shew in what manner the constancy of the ratio of the sines of incidence and refraction may be deduced from it.

15. To find the deviation of a transit instrument from the plane of the meridian by the observed superior and inferior transits of the same circumpolar star.

16. Shew that the calculations of the beginning and duration of an eclipse of the Moon, of an eclipse of the Sun, of an occultation of a fixed star by the Moon or a planet, and of the transit of Venus or Mercury over the Sun's disc, are, after proper modifications, reducible to the solution of the same problem.

17. Compare the time of the Moon's falling to the Earth by the action of gravity, with her periodic time.

18. When the force varies inversely as $(\text{dist.})^3$: from the requisite data, find the path of a projectile, the velocity being less than that in a circle at the same distance.

19. A particle is placed in the centre of a thin hemispherical shell, to every particle of which it is attracted by forces varying inversely as the square of its distance: determine the circumstances of its motion.

20. Given the length of a degree of the meridian in a given latitude, with the length of a degree $\perp$ to the meridian at the same place, to find the ratio of the polar and equatorial diameters.

21. Explain the physical causes of evection and variation of the Moon.

22. What is the peculiar use and advantage of Mercator's projection for the purposes of navigation.

23. What would be the effect of the Moon's action upon the Earth, supposing she had no rotation round her axis? Explain the change of this effect which is produced by her rotation.

TRINITY COLLEGE, 1827.

1. What is the solid content of a cube whose edge is 6 feet 6 inches.

2. To construct a ▱, which shall be equal to a given △ and have an ∠ equal to a given ∠.

3. Those △^{s} are similar, which have their sides about an equal angle in each proportionals.

4. Solve the equation

$$x^2 + \frac{36}{x^2} - 2x - \frac{12}{x} = 3.$$

5. Find integral values of x and y in the equation

$$7x + 19y = 73.$$

6. Given two sides and the included angle of a plane △: investigate and adapt to logarithmic computation the proper formulæ for finding the remaining side and angles.

7. Define a differential, and apply it to determine the differential of a^x.

8. Investigate the relation between the power and the weight upon the inclined plane.

9. In different ellipses round the same focus, the periodic times are in the sesquiplicate ratio of the major axes.

10. The volume of a sphere is $\frac{2}{3}$ of its circumscribing cylinder.

TRINITY COLLEGE, Sept. 1828.

1. Draw from the vertex of a triangle a straight line which shall be a mean proportional between the segments of the base.

How many answers does this question admit of?

2. Solve the equation $x^3 + x = 500$, and find a root to two places of decimals.

How many possible roots are there?

3. Find the centre of gravity of a frustum of a cone. Shew the position to which it tends as the ends tend to equality.

4. If L describes about T areas proportional to the times, L is acted upon by a force which tends to T.

Explain particularly the case where T is in motion.

5. If $y \propto x^m$ in one curve and $y \propto x^n$ in another, m being greater than n, shew that the first curve always falls between the other and its tangent in the immediate neighbourhood of the vertex.

6. Corporis in datâ trajectoriâ parabolicâ moti, invenire locum ad tempus assignatum.

7. Shew how we may find by observation the place and time of the equinox.

8. When a body descends in a fluid, find the greatest velocity which it can acquire. Hence knowing the rate at which a powder descends in a fluid, find the size of its particles.

9. A particle moves upon a given surface acted upon by no force. Determine its motion. What will be the path in the case of a cone?

10. By three observations of a solar spot determine the position of the Sun's axis of revolution.

11. If a ray diverging from the origin of co-ordinates x, y falls upon the curve, the distance to the point where it cuts the axis after reflexion will be $= \dfrac{2(x+py)(px-y)}{2px-(1-p^2)y}$, where $p = \dfrac{dy}{dx}$.

12. In a triangle the continued product of the four radii of the circles of contact is equal to the square of the area of the triangle.

13. Given the length of a curve, to find its form that the centre of gravity may be the lowest possible.

TRINITY COLLEGE, SEPT. 1828.

1. If a straight line falls on two parallel straight lines, it makes the alternate angles equal to one another.

What is the difficulty of proving this proposition satisfactorily? Mention any of the ways by which mathematicians have endeavoured to get over this difficulty.

2. Two points are taken on a wall and joined by a line which passes round a corner of the wall. This line is the shortest when its parts make equal angles with the edge at which the parts of the wall meet.

3. Prove the formulæ

$$\tan.\left(\frac{\pi}{4} \pm \frac{A}{2}\right) = \sec.A \pm \tan.A,$$

$$2 \tan.^{-1}\tfrac{1}{3} + \tan.^{-1}\tfrac{1}{7} = \frac{\pi}{4}.$$

4. From the equation $y = \frac{x}{1 + x^3}$ draw the curve. Find the angle at which it cuts the axis, and whether the area from $x = 0$ to $x = \text{inf.}$ is finite or infinite.

5. If $y = \cos.x$ prove that

$$\cos.mx = \pm \left\{ 1 - \frac{m^2y^2}{1.2} + \frac{m^2(m^2-4)}{1.2.3.4} - \&c. \right\} \quad (m \text{ even}),$$

$$\cos.mx = \pm \left\{ my - \frac{m(m^2-1)}{1.2} y^3 + \&c. \right\} \quad (m \text{ odd}).$$

What is in each case the last term and the number of terms? Write down the expression for $\cos.mx$ in terms of y when m is *fractional.*

6. The equation $ax^2 - by^2 + 2cz = 0$ belongs to a curve surface. What is the nature of this surface? In what direction is its concavity? Draw a normal to a given point of it.

7. When three forces keep a body at rest, what are the conditions which they fulfil?

Hence find the conditions for the equilibrium of the voussoirs of an arch. How are these affected by taking into account the friction of contiguous voussoirs?

8. A bowl filled with quicksilver is placed in an inverted position on a horizontal plane: what must be its weight that the fluid may not escape?

If there be a portion of air above the fluid, how will this affect the answer?

9. Two inclined planes being at right angles, a uniform straight beam rests between them. Shew that its middle point is highest when it makes with each plane an angle equal to the inclination of the other plane.

10. A candle is placed before a convex lens at twice its focal distance. At what distance must a plane be placed to receive the image of the candle? And what will be the form of the illuminated part at any other distance?

11. The equation of time is nothing four times in the year.

12. S is a fixed point, P a point in a curve, PT the tangent meeting a fixed line SA in T: to determine the curve from the property $SP = m \times ST$. What is the curve when $m = 1$?

13. Find the path of a body projected with a given velocity and acted on by a force which is as $\frac{1}{(\text{dist.})^3}$.

Why does Newton's method not give all the cases?

14. When P revolves about T, disturbed by S, the eccentricity of P's orbit increases from the time of its apsides being in quadratures to the time of the apsides being in syzygies.

What is the name and nature of the inequality of the Moon's motion thus produced?

15. Four equal particles placed in a straight line repel each other with forces varying inversely as any power of their distance. The two extreme ones being immoveable, the two intermediate ones perform *small* oscillations in the straight line. Define their motion.

16. A pendulum which oscillates n times a day at the Earth's surface, is carried to a depth z below the surface, and there gains t seconds a day. The radius of the earth being 1, shew that its mean density is

$$\frac{3}{2\left(1 - \dfrac{t}{nz}\right)}.$$

17. Find the difference of the asymptote and arc of an hyperbola when both are infinite.

18. If a, b be the semi-axes of an ellipse revolving in its own plane, M its mass, and Mk its moment of inertia, $k^2 = \dfrac{a^2 + b^2}{4}$.

19. On a spheroid, if l be the latitude, λ the *angle at the centre*, and $m = \dfrac{a^2 - b^2}{a^2 + b^2}$, we shall have

$$l = \lambda - m \sin.2\lambda + \frac{m^2}{2} \sin.4\lambda - \frac{m^3}{3} \sin.6\lambda, \text{ \&c.}$$

20. *A*'s skill is to *B*'s as $2 : 1$ $(m : n)$. *A* wants 6 (p) games to win and *B* wants 3 (q): what are their chances respectively?

21. Find the Moon's variation. What is the *argument* of this inequality?

22. Explain and prove the *principle of least action.*

TRINITY COLLEGE, 1830.

1. The price of gold is £3 17*s*. 10½*d*. an ounce, and a cubic inch weighs 10 ounces. What would be the cost of gilding the surface of England (a triangle of which the base and the perpendicular are each 315 miles) with gold ·00019 of an inch thick.

2. *A* sets off from Cambridge to London, and 2½ hours afterwards *B* starts from London to Cambridge: after 3 hours more they meet; and *A* reaches London 3¼ hours sooner than *B* reaches Cambridge. In what time does each perform the journey?

3. Give those geometrical definitions which are *used* in the proof of propositions. What other definitions would be requisite to supersede the axioms entirely? Prove *Euclid,* I. 4; and explain why the corresponding proposition concerning spherical triangles cannot be proved in the same manner?

4. While sailing S.W. I observe two ships at anchor, one at N.N.W. and the other at W.N.W. After running 5 miles these ships are seen at N. by W. and N.W. respectively. Required their bearing and distance from each other?

5. A pyramid has for its base an equilateral triangle of which each side is 1 foot, and its slant edge is 3 feet. Required the angles which its faces make with each other.

6. A weight slides on a string which is without inertia, and has its extremities fixed: (1) find the position of rest when there is no friction: (2) find the friction requisite to sustain the weight in any other position: (3) when the weight slides freely, find the equations of motion.

7. Show that the centre of pressure of a plane coincides with its centre of percussion: the axis of motion being the intersection of the plane with the surface of the fluid.

8. Find the focus of a refracting sphere, the index of refraction being 1·9. The diameter of the sphere being $\frac{1}{20}$ of an inch, what is the breadth of lines which through it appears like lines of $\frac{1}{10}$ of an inch breadth seen with the naked eye at 10 inches?

9. What is the use of observations of known stars made with a transit instrument: and the manner of making them? Prove the following formula for finding the deviation of a transit instrument from the meridian by means of a high and a low star:

$$(\tau - t)\frac{\cos.\delta \cos.\delta'}{\cos.l \sin.(\delta - \delta')}.$$

In which τ is the true difference of right ascension, and t the observed difference; δ, δ', l the declinations and latitude. Why are a *high* and *low* star selected?

10. Prove the following theorem; if $y = 2\cos.x$,

$$\cos.mx = \pm\left\{1 - \frac{m^2}{1.2}y^2 + \frac{m^2(m^2-4)}{1.2.3.4}y^4 - \&c.\right\}$$

according as m is even or odd.

Also give and prove the formula when m is fractional.

11. Find the evolute of the common parabola, and show where it cuts the original curve.

12. In the equation

$$y = x\sqrt{\frac{1-e^2x^2}{1-x^2}},$$

when e is less than 1, trace the curve, and find its area from

$$x = 0 \text{ to } x = 1.$$

13. Let ABC be a spherical triangle, in which C and c are constant: and let $e = \frac{\sin.C}{\sin.c}$. Then prove

$$\frac{da}{\sqrt{(1-e^2\sin.^2a)}} = \frac{db}{\sqrt{(1-e^2\sin.^2b)}};$$

and show that the integral of this equation is

$$\cos.a\cos.b + \sqrt{(1-e^2\sin.^2c)}\sin.a\sin.b = \cos.c.$$

14. "All the particles of matter gravitate to each other with forces which are inversely as the square of the distance." State the steps by which Newton establishes this proposition, and their connection.

15. When a small pencil of rays is refracted obliquely at a spherical surface, find the primary and secondary focal lines; and explain why they do not coincide.

16. Having given the weight and velocity of a hammer with which a nail is driven into a piece of wood by one blow, find the force requisite to draw it out: the resistance being supposed equal both ways.

17. If a fluid move *steadily* (*i. e.* so that the motion in the same parts of space is always the same) prove that

$$p - p_1 = P - \tfrac{1}{2}(v^2 - v_1^2),$$

p being the pressure at any point where the velocity is v; p_1 the pressure at a point of the surface, and v_1 the velocity of a particle there; and P being the pressure produced by the same forces in a fluid at rest: the density being 1.

18. What is meant by a *developable* surface? Investigate its analytical property.

19. State the optical theories of *emanation* and of *undulation*. What facts will both explain, and what one only? What is the principle of interferences, and how does it account for the fringes which accompany the shadows of minute bodies?

20. Prove the equation of the Moon's motion,

$$\frac{d^2u}{d\theta^2} + u - \frac{\frac{P}{u^2} - \frac{T}{u^3}\frac{du}{d\theta}}{h^2 + 2\int_\theta \frac{T}{u^3}} = 0.$$

What is the next step in the investigation?

21. In planets which are oblate by their revolution, the ellipticity varies inversely as the density × the square of the time of revolution.

22. A's skill is to B's as 3 : 2; A wants 3 games, B wants 2; what are their respective chances of winning?

23. The places of the stars are to be represented by projecting them as they appear to an eye at the centre, on a *cube* circumscribing the sphere. Give the formulæ for the place of any star.

THE END.

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